

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
June 2010

Mathematics

MPC4

Unit Pure Core 4

Tuesday 15 June 2010 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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7	
8	
TOTAL	



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Answer **all** questions in the spaces provided.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 8x^3 + 6x^2 - 14x - 1$.

Find the remainder when $f(x)$ is divided by $(4x - 1)$. (2 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 8x^3 + 6x^2 - 14x + d$.

(i) Given that $(4x - 1)$ is a factor of $g(x)$, find the value of the constant d . (2 marks)

(ii) Given that $g(x) = (4x - 1)(ax^2 + bx + c)$, find the values of the integers a , b and c . (3 marks)

QUESTION
PART
REFERENCE



QUESTION
PART
REFERENCE

Turn over ►



2 A curve is defined by the parametric equations

$$x = 1 - 3t, \quad y = 1 + 2t^3$$

- (a) Find $\frac{dy}{dx}$ in terms of t . (3 marks)
- (b) Find an equation of the normal to the curve at the point where $t = 1$. (4 marks)
- (c) Find a cartesian equation of the curve. (2 marks)

QUESTION
PART
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QUESTION
PART
REFERENCE

Turn over ►



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3 (a) (i) Express $\frac{7x-3}{(x+1)(3x-2)}$ in the form $\frac{A}{x+1} + \frac{B}{3x-2}$. (3 marks)

(ii) Hence find $\int \frac{7x-3}{(x+1)(3x-2)} dx$. (2 marks)

(b) Express $\frac{6x^2+x+2}{2x^2-x+1}$ in the form $P + \frac{Qx+R}{2x^2-x+1}$. (3 marks)

QUESTION
PART
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QUESTION
PART
REFERENCE

Turn over ►



4 (a) (i) Find the binomial expansion of $(1+x)^{\frac{3}{2}}$ up to and including the term in x^2 .
(2 marks)

(ii) Find the binomial expansion of $(16+9x)^{\frac{3}{2}}$ up to and including the term in x^2 .
(3 marks)

(b) Use your answer to part **(a)(ii)** to show that $13^{\frac{3}{2}} \approx 46 + \frac{a}{b}$, stating the values of the integers a and b .
(2 marks)

QUESTION
PART
REFERENCE



[illegible]

$$3 \sin^2 x - \sin x - 2 = 0 \quad (3 \text{ marks})$$

(ii) Hence, given that $3 \cos 2x + 2 \sin x + 1 = 0$, find the possible values of $\sin x$.
(2 marks)

(b) (i) Express $3 \cos 2x + 2 \sin 2x$ in the form $R \cos(2x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving α to the nearest 0.1° . (3 marks)

(ii) Hence solve the equation

$$3 \cos 2x + 2 \sin 2x + 1 = 0$$

for all solutions in the interval $0^\circ < x < 180^\circ$, giving x to the nearest 0.1° .
(3 marks)

[illegible]

[illegible]

6 A curve has equation $x^3y + \cos(\pi y) = 7$.

(a) Find the exact value of the x -coordinate at the point on the curve where $y = 1$. (2 marks)

(b) Find the gradient of the curve at the point where $y = 1$. (5 marks)

QUESTION
PART
REFERENCE



[illegible]

7 The point A has coordinates $(4, -3, 2)$.

The line l_1 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$.

The point B lies on l_2 where $\mu = 2$.

(a) Find the vector $\overrightarrow{AB}$. (3 marks)

(b) (i) Show that the lines l_1 and l_2 intersect. (4 marks)

(ii) The lines l_1 and l_2 intersect at the point P . Find the coordinates of P . (1 mark)

(c) The point C lies on a line which is parallel to l_1 and which passes through the point B . The points A, B, C and P are the vertices of a parallelogram.

Find the coordinates of the two possible positions of the point C . (4 marks)

QUESTION
PART
REFERENCE



[illegible]

- 8 (a) Solve the differential equation

$$\frac{dx}{dt} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

given that $x = 80$ when $t = 0$. Give your answer in the form $x = f(t)$. (6 marks)

- (b) A fungus is spreading on the surface of a wall. The proportion of the wall that is unaffected after time t hours is $x\%$. The rate of change of x is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{1}{5}(x+1)^{\frac{1}{2}}$$

At $t = 0$, the proportion of the wall that is unaffected is 80%. Find the proportion of the wall that will still be unaffected after 60 hours. (2 marks)

- (c) A biologist proposes an alternative model for the rate at which the fungus is spreading on the wall. The total surface area of the wall is 9 m^2 . The surface area that is **affected** at time t hours is $A \text{ m}^2$. The biologist proposes that the rate of change of A is proportional to the product of the surface area that is affected and the surface area that is unaffected.

- (i) Write down a differential equation for this model.

(You are not required to solve your differential equation.) (2 marks)

- (ii) A solution of the differential equation for this model is given by

$$A = \frac{9}{1 + 4e^{-0.09t}}$$

Find the time taken for 50% of the area of the wall to be affected. Give your answer in hours to three significant figures. (4 marks)

QUESTION
PART
REFERENCE



[illegible]

[illegible]

[illegible]

There are no questions printed on this page

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ANSWER IN THE SPACES PROVIDED**

