Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MD02

Unit Decision 2

Monday 20 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

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Answer **all** questions in the spaces provided.

1 **Figure 1** below shows an activity diagram for a cleaning project. The duration of each activity is given in days.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion of the project. *(3 marks)*

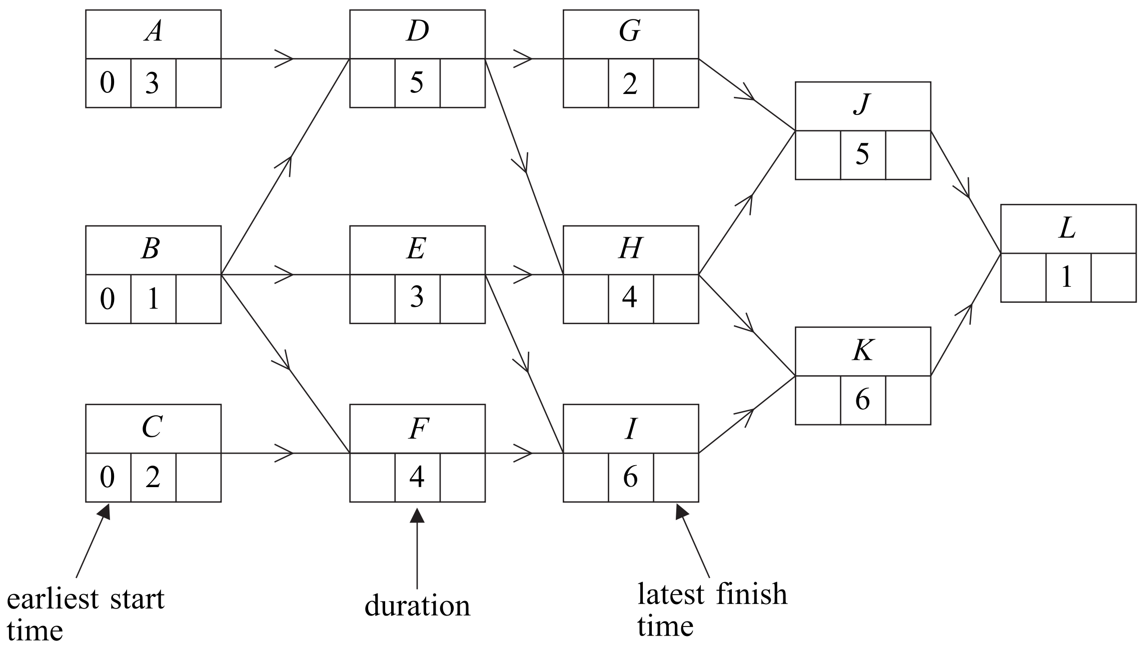
(c) Find the activity with the greatest float time and state the value of its float time. *(2 marks)*

(d) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as **late** as possible. *(4 marks)*

QUESTION PART REFERENCE

(a)

Figure 1



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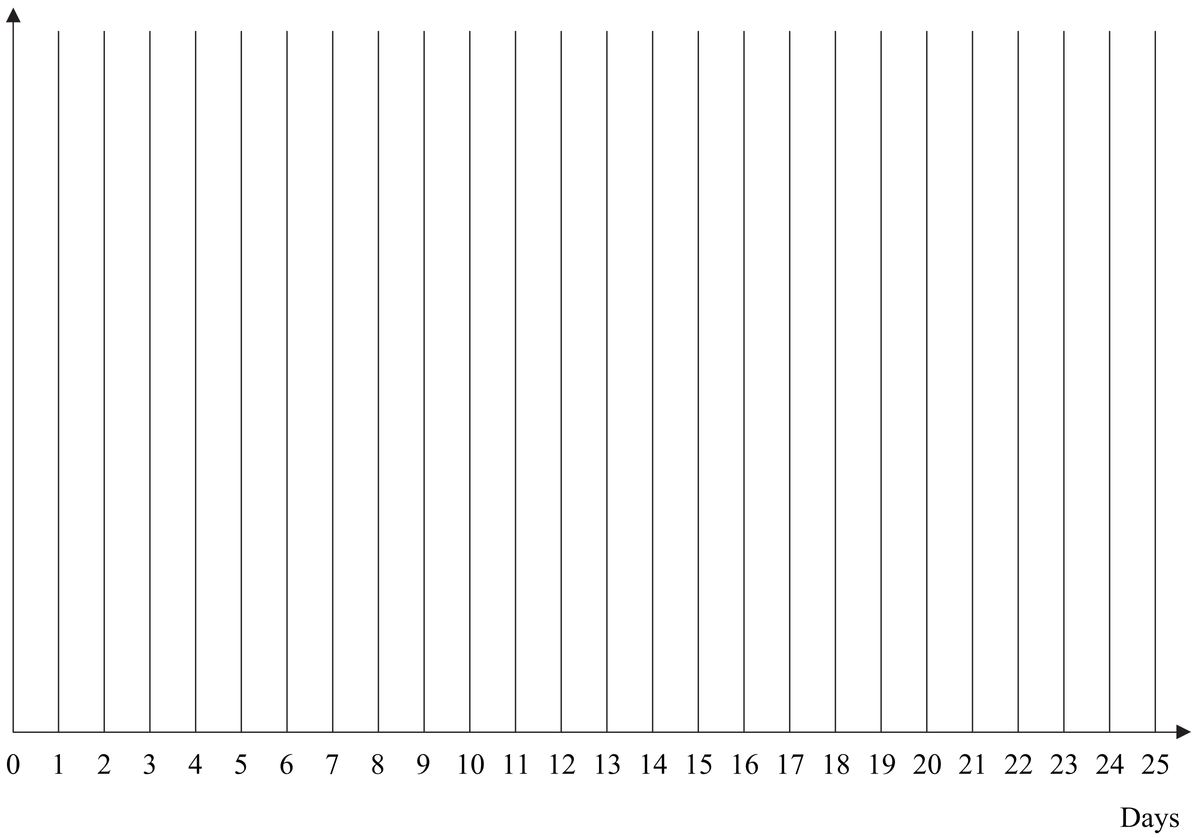
(d)

Figure 2



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2 The times taken, in minutes, for five people, A, B, C, D and E, to complete each of five different puzzles are recorded in the table below.

	A	B	C	D	E
Puzzle 1	16	13	15	16	15
Puzzle 2	14	16	16	14	18
Puzzle 3	14	12	18	13	16
Puzzle 4	15	15	17	12	14
Puzzle 5	13	17	16	14	15

Using the Hungarian algorithm, each of the five people is to be allocated to a different puzzle so that the total time for completing the five puzzles is minimised.

(a) By reducing the **columns first** and then the rows, show that the new table of values is

3	1	0	4	1
0	k	0	1	3
1	0	3	1	2
2	3	2	0	0
0	5	1	2	1

State the value of the constant k. *(2 marks)*

(b) (i) Show that the zeros in the table in part **(a)** can be covered with one horizontal and three vertical lines. *(1 mark)*

(ii) Use augmentation to produce a table where five lines are required to cover the zeros. *(2 marks)*

(c) Hence find all the possible ways of allocating the five people to the five puzzles so that the total completion time is minimised. *(3 marks)*

(d) Find the minimum total time for completing the five puzzles. *(1 mark)*

(e) Explain how you would modify the original table if the Hungarian algorithm were to be used to find the **maximum** total time for completing the five puzzles using five different people. *(1 mark)*

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3 (a) Two people, Tom and Jerry, play a zero-sum game. The game is represented by the following pay-off matrix for Tom.

Jerry

	Strategy	**A**	**B**	**C**
	I	-4	5	-3
Tom	**II**	-3	-2	8
	III	-7	6	-2

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Rohan and Carla play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Rohan.

Carla

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Rohan	$\mathbf{R_1}$	3	5	-1
	$\mathbf{R_2}$	1	-2	4

(i) Find the optimal mixed strategy for Rohan and show that the value of the game is $\frac{3}{2}$. *(7 marks)*

(ii) Carla plays strategy C_1 with probability p, and strategy C_2 with probability q.

Find the values of p and q and hence find the optimal mixed strategy for Carla. *(4 marks)*

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4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 6y + kz$, where k is a constant.

The initial Simplex tableau is given below.

P	x	y	z	s	t	u	*value*
1	-2	-6	$-k$	0	0	0	0
0	5	3	10	1	0	0	15
0	7	6	4	0	1	0	28
0	4	3	6	0	0	1	12

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot **from the *y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value has **not** been reached, find the possible values of k. *(2 marks)*

(c) In the case when $k = 20$:

(i) perform one further iteration; *(4 marks)*

(ii) interpret the final tableau and state the values of the slack variables. *(3 marks)*

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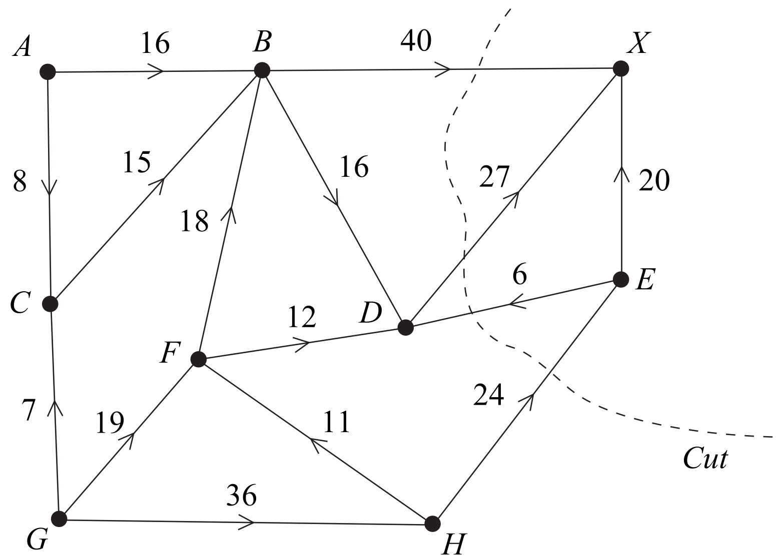
5 The network shows the evacuation routes along corridors in a college, from two teaching areas to the exit, in case of a fire alarm sounding.



The two teaching areas are at A and G and the exit is at X.

The number on each edge represents the maximum number of people that can travel along a particular corridor in one minute.

(a) Find the value of the cut shown on the diagram. *(1 mark)*

(b) Find the maximum flow along each of the routes $ABDX$, $GFBX$ and $GHEX$ and enter their values in the table on **Figure 3** opposite. *(3 marks)*

(c) (i) Taking your answers to part **(b)** as the initial flow, use the labelling procedure on **Figure 3** to find the maximum flow through the network. You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow, and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(d) During one particular fire drill, there is an obstruction allowing no more than 45 people per minute to pass through vertex B. State the maximum number of people that can move through the network per minute during this fire drill. *(2 marks)*

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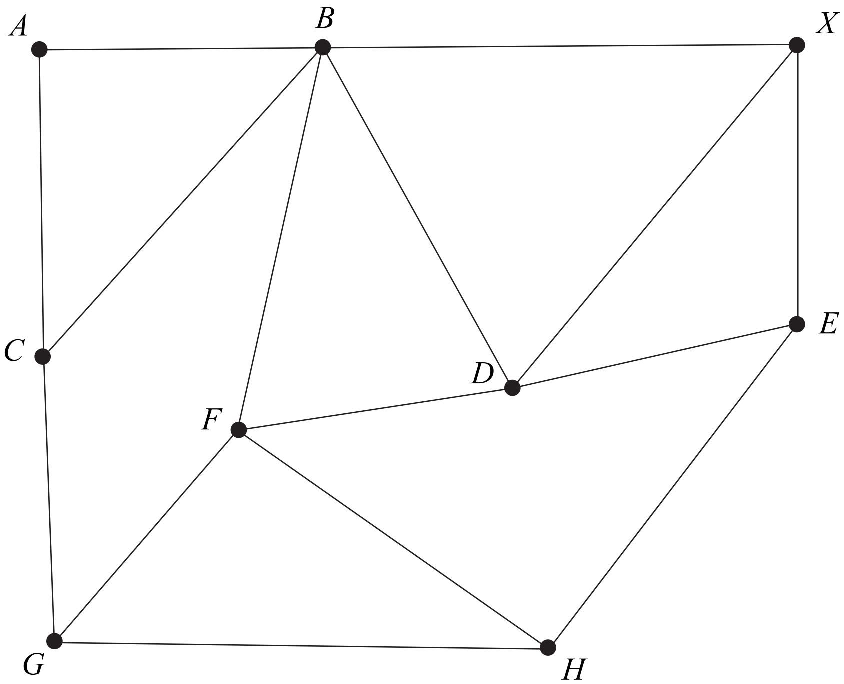
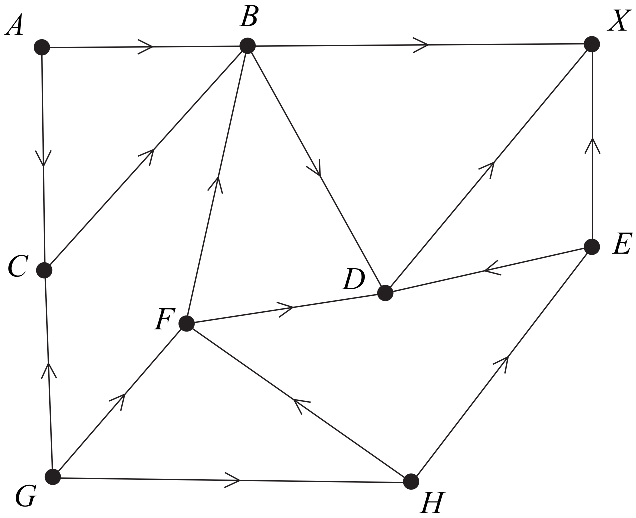
Figure 3



Route	Flow
ABDX	
GFBX	
GHEX	

Figure 4



Maximum flow is .... people per minute.

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6 Bob is planning to build four garden sheds, A, B, C and D, at the rate of one per day. The order in which they are built is a matter of choice, but the costs will vary because some of the materials left over from making one shed can be used for the next one. The expected profits, in pounds, are given in the table below.

Day	Already built	Expected profit (£)			
		A	B	C	D
Monday	–	50	65	70	80
Tuesday	A	–	72	83	84
	B	60	–	80	83
	C	57	68	–	85
	D	62	70	81	–
Wednesday	A and B	–	–	84	88
	A and C	–	71	–	82
	A and D	–	74	83	–
	B and C	65	–	–	86
	B and D	69	–	85	–
	C and D	66	73	–	–
Thursday	A, B and C	–	–	–	90
	A, B and D	–	–	87	–
	A, C and D	–	76	–	–
	B, C and D	70	–	–	–

By completing the table of values opposite, or otherwise, use dynamic programming, **working backwards from Thursday**, to find the building schedule that maximises the total expected profit. *(9 marks)*

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Stage (Day)	State (Sheds already built)	Action (Shed to build)	Calculation	Profit in pounds
Thursday	A, B, C	D		90
	A, B, D	C		87
	A, C, D	B		76
	B, C, D	A		70
Wednesday	A, B	C	$84 + 90$	174
		D	$88 + 87$	175
	A, C	B	$71 + 90$	161
		D	$82 + 76$	158
	A, D	B		
		C		
	B, C	A		
		D		
	B, D	A		
		C		
	C, D	A		
		B		
Tuesday	A	B	$72 + 175$	247
		C	$83 + 161$	244
		D		
Monday				

Schedule

	Monday	Tuesday	Wednesday	Thursday
Shed to build				

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