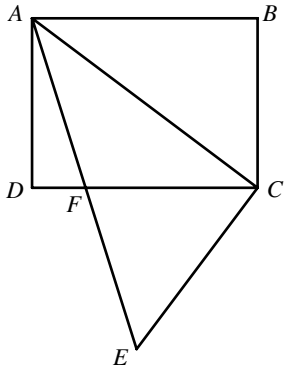
13. A manager in a store has to determine the price of a sweater. Market research gives him the following data: If the price is €75, then 100 teenagers will buy the sweater. Each time the price is increased by €5, 20 fewer teenagers will buy the sweater. However, each time the price is decreased by €5, 20 sweaters more will be sold. The sweaters cost the company €30 apiece.
What is the sale price that maximizes profits?

14. The diagram shows a rectangle $ABCD$ with $AB = 16$ and $BC = 12$. Angle ACE is a right angle and $CE = 15$. The line segments AE and CD meet at F.
What is the area of triangle ACF?



15. For each real number x, let $f(x)$ be the minimum of the numbers $3x + 1$, $2x + 3$ and $-4x + 24$.
What is the maximum value of $f(x)$?

16. The integer m has ninety-nine digits, all of them nines.
What is the sum of the digits of m^2 ?

17. In rectangle $ABCD$, the midpoints of sides BC, CD and DA are P, Q and R respectively. The point M is the midpoint of QR. The area of triangle APM is a fraction m/n of the area of rectangle $ABCD$, where m and n are integers and m/n is in its simplest form.
What is the value of $m + n$?

18. The integers a, b and c are such that $0 < a < b < c < 10$. The sum of all three-digit numbers that can be formed by a permutation of these three integers is 1554.
What is the value of c?

19. Given that $\left(a + \frac{1}{a}\right)^2 = 6$ and $a^3 + \frac{1}{a^3} = N\sqrt{6}$ and $a > 0$, what is the value of N?

20. The polynomial $f(x)$ is such that $f\left(x^2 + 1\right) \equiv x^4 + 4x^2$ and $f\left(x^2 - 1\right) \equiv ax^4 + 4bx^2 + c$.
What is the value of $a^2 + b^2 + c^2$?

SENIOR 'KANGAROO' MATHEMATICAL CHALLENGE

Friday 2nd December 2011

Organised by the United Kingdom Mathematics Trust

The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.

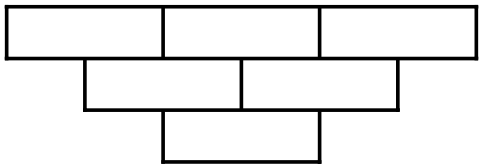
RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.
   Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.
5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

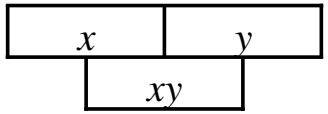
Enquiries about the Senior Kangaroo should be sent to:
Maths Challenges Office, School of Maths Satellite,
University of Leeds, Leeds, LS2 9JT
Tel. 0113 343 2339
www.ukmt.org.uk

1. The diagram below is to be completed so that:
   a) each cell contains a positive integer;
   b) apart from the top row, the number in each cell is the product of the numbers in the two cells immediately above;
   c) the six numbers are all different.

   What is the smallest possible total of the six numbers?



RULE:



2. The mean number of students accepted by a school in the four years 2007 to 2010 was 325. The mean number of students accepted by the school in the five years 2007 to 2011 was 4% higher.

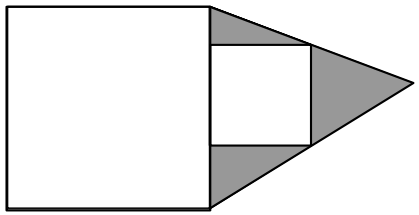
   How many students did this school accept in 2011?

3. 200 people stand in a line. The prize-giver walks along the line 200 times, always starting at the same end. On the first pass, the prize-giver gives each person a pound coin. On the second pass along the line, the prize-giver gives every second person another pound. On the third pass, every third person is given another pound, and so on.

   After 200 passes, how many pounds has the 120th person been given?

4. The diagram below includes two squares: one has sides of length 20 and the other has sides of length 10.

   What is the area of the shaded region?



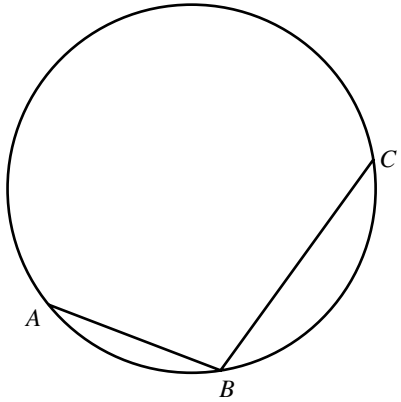
5. How many positive two-digit numbers are there whose square and cube both end in the same digit?

6. The lengths of two sides of an acute-angled triangle and the perpendicular height from the third side of the triangle are 12, 13 and 15 (possibly not in that order).

   What is the area of the triangle?

7. In the diagram, the radius of the circle is equal to the length AB.

   What is the size of angle ACB, in degrees?



8. The price of an item in pounds and pence is increased by 4%. The new price is exactly n pounds where n is a whole number.

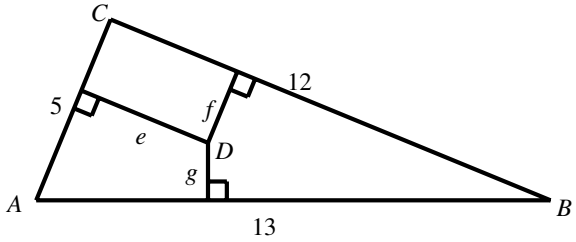
   What is the smallest possible value of n?

9. How many squares have $(-1, -1)$ as a vertex and at least one of the coordinate axes as an axis of symmetry?

10. What is the value of $\left(\sqrt{8 + 2\sqrt{7}} - \sqrt{8 - 2\sqrt{7}}\right)^2$?

11. In the diagram, ABC is a triangle with sides $AB = 13$, $BC = 12$ and $AC = 5$. The point D is any point inside the triangle with $CD = 4$ and the perpendicular distances from D to the sides of the triangle are e, f and g, as shown.

    What is the value of $5e + 12f + 13g$?



12. Elections in Herbville were held recently. Everyone who voted for the Broccoli Party had already eaten broccoli. Of those who voted for other parties, 90% had never eaten broccoli. Of those who voted, 46% had eaten broccoli.

    What percentage voted for the Broccoli Party?