UK Junior Mathematical Olympiad 2012

Organised by The United Kingdom Mathematics Trust

Tuesday 12th June 2012

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 45 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

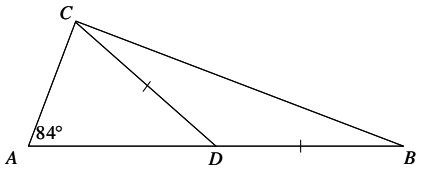
Section A

A1 What is the value of $1^1 + 2^2 + 3^3 + 4^4 - (1^4 + 2^3 + 3^2 + 4^1)$?

A2 Mike drank 60% of his glass of milk. Afterwards, 80 ml of milk remained in the glass. What volume of milk was initially in the glass?

A3 In triangle ABC, $\angle CAB = 84°$; D is a point on AB such that $\angle CDB = 3 \times \angle ACD$ and $DC = DB$.

What is the size of $\angle BCD$?

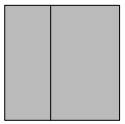


A4 A book costs £3.40 and a magazine costs £1.60. Clara spends exactly £23 on books and magazines. How many magazines does she buy?

A5 Each digit of a positive integer is 1 or 2 or 3.
Given that each of the digits 1, 2 and 3 occurs at least twice, what is the smallest such integer that is not divisible by 2 or 3?

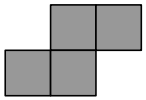
A6 A square is cut into two rectangles, as shown, so that the sum of the lengths of the perimeters of these two rectangles is 30 cm.

What is the length of a side of the square?



A7 The diagram shows a shape made from four 1 × 1 squares.

What is the maximum number of such shapes that can be placed inside a 5 × 5 square without overlapping?
(The shapes may be rotated or turned over.)



A8 An athletics club has junior (i.e. boy or girl) members and adult members. The ratio of girls to boys to adults is 3 : 4 : 9 and there are 16 more adult members than junior members. In total, how many members does the club have?

A9 What is the integer x so that $\frac{x}{9}$ lies between $\frac{71}{7}$ and $\frac{113}{11}$?

A10 A positive integer, N, has three digits and the product of its digits is also a three-digit integer. What is the smallest possible value of N?

Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

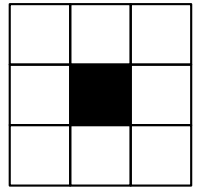
B1 There was an old woman who lived in a shoe. She had 9 children at regular intervals of 15 months. The oldest is now six times as old as the youngest. How old is the youngest child?

B2 Anastasia thinks of a positive integer, which Barry then doubles. Next, Charlie trebles Barry's number. Finally, Damion multiplies Charlie's number by six. Eve notices that the sum of these four numbers is a perfect square. What is the smallest number that Anastasia could have thought of?

B3 Mr Gallop has two stables which each initially housed three ponies. His prize pony, Rein Beau, is worth £250 000. Usually Rein Beau spends his day in the small stable, but when he wandered across into the large stable, Mr Gallop was surprised to find that the average value of the ponies in each stable rose by £10 000. What is the total value of all six ponies?

B4 An irregular pentagon has five different interior angles each of which measures an integer number of degrees. One angle is 76°.
The other four angles are three-digit integers which fit one digit per cell across and down into the grid on the right.

In how many different ways can the grid be completed?



B5 Three identical, non-overlapping, squares $ABCD$, $AEFG$, $AHIJ$ (all labelled anticlockwise) are joined at the point A, and are 'equally spread' (so that $\angle JAB = \angle DAE = \angle GAH$). Calculate $\angle GBH$.

B6 The integer 23173 is such that

(a) every pair of neighbouring digits, taken in order, forms a prime number;
and (b) all of these prime numbers are different.

What is the largest integer which meets these conditions?