

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
January 2012

Mathematics

MPC4

Unit Pure Core 4

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
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TOTAL	



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Answer **all** questions in the spaces provided.

1 (a) Express $\frac{2x+3}{4x^2-1}$ in the form $\frac{A}{2x-1} + \frac{B}{2x+1}$, where A and B are integers. (3 marks)

(b) Express $\frac{12x^3 - 7x - 6}{4x^2 - 1}$ in the form $Cx + \frac{D(2x+3)}{4x^2 - 1}$, where C and D are integers. (3 marks)

(c) Evaluate $\int_1^2 \frac{12x^3 - 7x - 6}{4x^2 - 1} dx$, giving your answer in the form $p + \ln q$, where p and q are rational numbers. (5 marks)

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QUESTION
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2 Angle α is acute and $\cos \alpha = \frac{3}{5}$. Angle β is **obtuse** and $\sin \beta = \frac{1}{2}$.

(a) (i) Find the value of $\tan \alpha$ as a fraction. (1 mark)

(ii) Find the value of $\tan \beta$ in surd form. (2 marks)

(b) Hence show that $\tan(\alpha + \beta) = \frac{m\sqrt{3} - n}{n\sqrt{3} + m}$, where m and n are integers. (3 marks)

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- 3 (a)** Find the binomial expansion of $(1 + 6x)^{\frac{2}{3}}$ up to and including the term in x^2 .
(2 marks)
- (b)** Find the binomial expansion of $(8 + 6x)^{\frac{2}{3}}$ up to and including the term in x^2 .
(3 marks)
- (c)** Use your answer from part **(b)** to find an estimate for $\sqrt[3]{100}$ in the form $\frac{a}{b}$, where a and b are integers.
(2 marks)

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- 4** A scientist is testing models for the growth and decay of colonies of bacteria.

For a particular colony, which is growing, the model is $P = Ae^{\frac{1}{8}t}$, where P is the number of bacteria after a time t minutes and A is a constant.

- (a) This growing colony consists initially of 500 bacteria. Calculate the number of bacteria, according to the model, after one hour. Give your answer to the nearest thousand. (2 marks)
- (b) For a second colony, which is decaying, the model is $Q = 500\,000e^{-\frac{1}{8}t}$, where Q is the number of bacteria after a time t minutes.

Initially, the growing colony has 500 bacteria and, at the same time, the decaying colony has 500 000 bacteria.

- (i) Find the time at which the populations of the two colonies will be equal, giving your answer to the nearest 0.1 of a minute. (3 marks)
- (ii) The population of the growing colony will exceed that of the decaying colony by 45 000 bacteria at time T minutes.

Show that

$$\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$$

and hence find the value of T , giving your answer to one decimal place. (4 marks)

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5

$$x = 8t^2 - t, \quad y = \frac{3}{t}$$

- (a)** Show that the cartesian equation of the curve can be written as $xy^2 + 3y = k$, stating the value of the integer k . (2 marks)
- (b) (i)** Find an equation of the tangent to the curve at the point P , where $t = \frac{1}{4}$. (7 marks)
- (ii)** Verify that the tangent at P intersects the curve when $x = \frac{3}{2}$. (2 marks)

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- are given by $\cos \theta = \frac{3}{4}$. (4 marks)

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7 Solve the differential equation

$$\frac{dy}{dx} = y^2 x \sin 3x$$

given that $y = 1$ when $x = \frac{\pi}{6}$. Give your answer in the form $y = \frac{9}{f(x)}$. (9 marks)

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The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$.

- (a) (i)** Find the vector $\overrightarrow{AB}$. (2 marks)
- (ii)** Find the acute angle between AB and the line l , giving your answer to the nearest degree. (4 marks)
- (b)** The point C lies on the line l such that the angle ABC is a right angle. Given that $ABCD$ is a rectangle, find the coordinates of the point D . (6 marks)

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