THE COLLEGES OF OXFORD UNIVERSITY

PHYSICS

Wednesday, 31 October 2007

Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

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You may use any calculator.
No tables may be used.

Take $g = 10\,\mathrm{ms}^{-2}$.

Attempt all 3 sections and as many questions as you can.

Marks for each question are indicated in the right hand margin.
There are a total of 50 marks available.

You are advised not to spend more than 15 minutes on Section A.

Do NOT turn over until told that you may do so.

Section A: multiple choice (10 marks)

1. A cube of metal has sides of length x. The electrical resistance between opposite faces of the cube is:

   A directly proportional to x **B** proportional to x^2
   C inversely proportional to x **D** independent of x [1]

2. An astronaut in the International space station experiences weightlessness because

   A She is outside the earth's gravitational field
   B The attractive force of the moon cancels that of the earth
   C She is moving
   D She is accelerating at the same rate as the space station [1]

3. A 9 V battery is connected across a 100 Ω resistor. Given that the charge on an electron is 1.6×10^{-19} C what is the number of electrons passing through the resistor every second?

   A 5.6×10^{17} **B** 6.9×10^{19}
   C 5.6×10^{21} **D** 6.9×10^{15} [1]

4. A drop slide in a fairground has a very steep initial slope which gradually curves into a more gentle slope. If a child drops down the slide, what happens to his speed v and the magnitude of his acceleration a?

   A v and a both increase
   B v increases and a stays the same
   C v increases and a decreases
   D v decreases and a increases [1]

5. A spring that obeys Hooke's law has a spring constant k. Two such springs are linked to form a spring of twice the length. What is the spring constant of this new longer spring?

   A $k/2$ **B** $k/\sqrt{2}$
   C $\sqrt{2}k$ **D** $2k$ [1]

6. Two resistors R_1 and R_2 are in parallel with a potential difference V across them. The total power dissipated in this circuit is

A	$V^2 \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$	**B**	$\frac{V^2}{R_1 + R_2}$
C	$V^2 \div \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$	**D**	$V^2 (R_1 + R_2)$

[1]

7. Positron Emission Tomography (PET) scanners frequently operate using the radioactive isotope ^{18}F, which has a half life of about two hours. The isotope is incorporated into a drug, half of which is excreted by the body every two hours. How long will it take before the quantity of radioactive drug in the body halves?

A	0.5 hours	**B**	1 hour
C	1.5 hours	**D**	2 hours

[1]

8. A pressure cooker has an escape valve that is essentially a 125 g weight resting on a circular hole of radius 1 mm. What pressure will lift the weight off the hole?

A	400 Pa	**B**	40 kPa
C	400 kPa	**D**	400 MPa

[1]

9. A car of mass 1500 kg is towing a trailer of mass 1000 kg at a steady speed. The driver decides to overtake another car and accelerates at 4 m/s^2. If the frictional force on the trailer is 2500 N what is the force on the towbar during the manoeuvre?

A	6500 N	**B**	8500 N
C	10000 N	**D**	12500 N

[2]

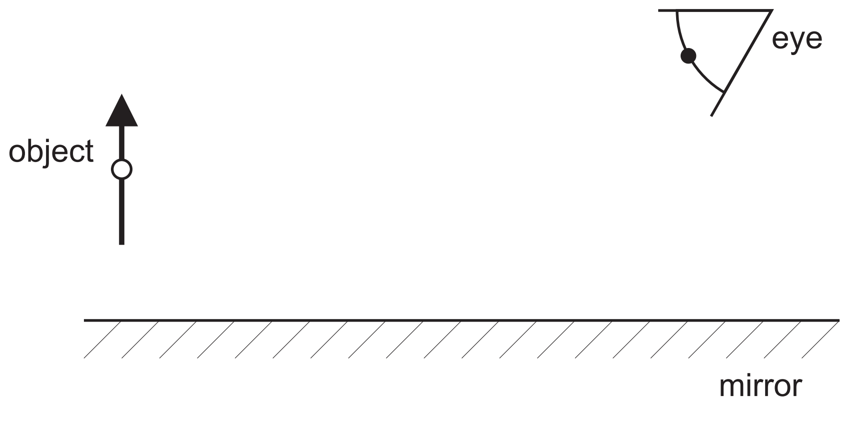
Section B: written answers (20 marks)

10. A pilot plans to fly an aeroplane from his base to another airport 300 km due north. His aeroplane can travel at a speed of 170 km/h relative to the air.

    (a) He sets off at 9:00 am and aims his aeroplane due north. Calculate his estimated arrival time assuming there is no wind. [2]

    (b) After flying for one hour he discovers that he is only 153 km from his starting point and has travelled along a line bearing 010°. Assuming the wind has been steady throughout the journey calculate its speed. [5]

11. Make a sketch copy of the diagram below and indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles. [3]



12. The planet Pluto (radius 1180 km) is populated by three species of purple caterpillar. Studies have established the following facts:

   (a) A line of 5 mauve caterpillars is as long as a line of 7 violet caterpillars

   (b) A line of 3 lavender caterpillars and 1 mauve caterpillar is as long as a line of 8 violet caterpillars.

   (c) A line of 5 lavender caterpillars, 5 mauve caterpillars and 2 violet caterpillars is 1 m long in total.

   (d) A lavender caterpillar takes 10 s to crawl the length of a violet caterpillar.

   (e) Violet and mauve caterpillars both crawl twice as fast as lavender caterpillars.

   How long would it take a mauve caterpillar to crawl around the equator of Pluto? [6]

13. The current in amperes through a certain type of non-linear resistor is given by $I = 0.05V^3$, where V is the potential difference in volts across the resistor. This resistor is connected in series to a fixed resistor and a constant voltage source of 9 V is connected across the series combination. What value of resistance should the fixed resistor have so that a current of 0.40 A flows? [4]

Section C: long question (20 marks)

14. You should already be familiar with the mathematical treatment of an ideal pendulum, in which the pendulum bob is modelled as a point mass on the end of a rigid rod of negligible mass. In this problem you will consider the behaviour of more complex types of pendulum. You will be given all the information you need in the sections below.

For a general pendulum of any shape and size the period P is given by

$$P = 2\pi\sqrt{\frac{I}{gML_{CM}}}$$

where g is the acceleration due to gravity, M is the total mass of the pendulum, L_{CM} is the effective length of the pendulum, defined as the distance from the pivot to the centre of mass, and I is the *moment of inertia* around the pivot point. For a point mass m fixed at a distance r from the pivot $I = mr^2$, while for a uniform rod of mass m and length r attached to the pivot at one end $I = \frac{1}{3}mr^2$. For more complex objects the total moment of inertia can be calculated by adding together values for the component parts.

(a) Identify L_{CM} and I for an ideal pendulum of length L with a bob of mass M, and hence calculate its period. [3]

(b) Repeat this calculation for a pendulum made from a unform rod of mass M and length L. [3]

(c) Now consider the case of a real pendulum, with a bob of mass M_b (which you may treat as a point mass) attached to the pivot using a uniform rod of mass M_r and length L, and find the period in this case. Show that the result for a real pendulum reduces to the results for an ideal pendulum and a rod pendulum by taking appropriate limits. [7]

For the remainder of this problem you can consider an ideal pendulum, for which you will calculate the effect of changing its environment.

(d) Most substances expand with increasing temperature, and so a metal rod will expand in length by a fraction $\alpha\,\delta T$ if the temperature is changed by δT, where α is called the coefficient of linear thermal expansion. Consider the effect of this expansion on a pendulum clock with a pendulum made from brass, with $\alpha = 19 \times 10^{-6}\,\mathrm{K}^{-1}$. What temperature change can this clock tolerate if it is to remain accurate to 1 second in 24 hours? [5]

(e) Repeat the calculation for a pendulum made from Invar alloy, with $\alpha = 1.2 \times 10^{-6}\,\mathrm{K}^{-1}$. [2]

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MATHEMATICS FOR PHYSICS

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1. Evaluate $6667^2 - 3333^2$ [2]

2. Find the equation of the line which is tangent to the curve $y = x^4$ at the point $(-2, 16)$. [3]

3. Evaluate

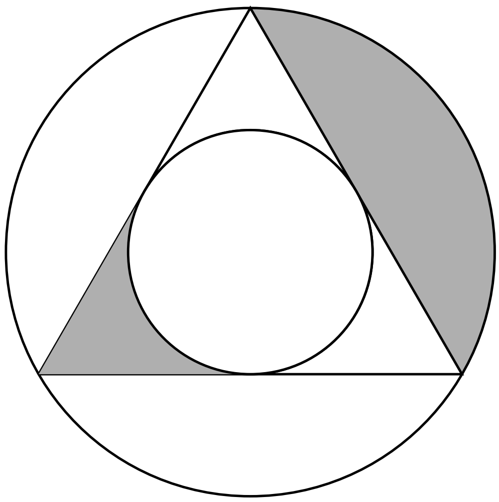
$$\frac{2 \log 125}{3 \log 25}$$

[3]

4. Two dice are thrown. What is the probability that their numbers add up to (i) six (ii) eleven? [4]

5. Expand $(2 + x)^5$ in ascending powers of x as far as the term in x^3. [3]

6. In the figure shown below, the triangle is equilateral. Find the ratio of the areas of
(i) the larger to the smaller circle;
(ii) the larger to the smaller shaded region. [6]



7. Sketch the curves $y = x^2$, $y = (x-2)^2$ and $y = x^2 + (x-2)^2$ on the same graph. [5]

8. Solve the equation $\tan\theta = 2\sin\theta$ for $0 \le \theta \le 2\pi$. [4]

9. Show that the points $(-5, 4)$, $(-1, -2)$ and $(5, 2)$ lie at three corners of a square. Find the coordinates of the fourth corner and the area of the square. [4]

10. Evaluate

$$\int_1^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \mathrm{d}x.$$

[4]

11. The first two terms of a geometric progression (first term a and common ratio r) are the same as the first two terms of an arithmetic progression (first term a and common difference d). The third term of the geometric progression is twice as big as the third term of the arithmetic progression. Find two different expressions for d and hence or otherwise find the two possible values of r. [6]

12. An isosceles triangle has sides of length x, x and $p - 2x$ where p is the length of the perimeter of the triangle. Find the value of x which maximises the area of the triangle for fixed p and all the angles of the triangle for this value of x.

[6]