

Kinematics

If the particle starts at O and moves away from it in a straight line, this direction is taken to be positive. If the particle moves towards O, the direction is taken to be negative.

$$\begin{array}{c} 0 \\ | \text{-----} \rightarrow +\text{ve direction} \\ s = 0 \end{array}$$

Displacement, s : distance from a fixed point O

Velocity and acceleration are examples of rates of change:

velocity, v : rate of change of **displacement**; $\pm$ indicates particle's direction.

acceleration, a : rate of change of velocity.

Example 1

A bicycle travels along a straight line. Its displacement, s m, from its starting position, O, after time t seconds is given by the equation

$$s = 4t + 3t^2 - t^3 \text{ m}$$

After a time of 2 seconds, find the bicycle's

a velocity in m/s

b acceleration in m/s^2

$$\mathbf{a} \quad s = 4t + 3t^2 - t^3$$

$$v = \frac{ds}{dt} = 4 + 6t - 3t^2 \text{ m/s}$$

$$\text{At } t = 2, v = 4 + 6(2) - 3(2)^2 \quad v = 4 \text{ m/s} \quad (\text{moving away from O})$$

$$\mathbf{b} \quad v = 4 + 6t - 3t^2$$

$$a = \frac{dv}{dt} = 6 - 6t \text{ m/s}^2$$

$$\text{At } t = 2, a = 6 - 6(2) \quad a = -6 \text{ m/s}^2 \quad (\text{slowing down})$$



KEY POINTS

- Velocity is the rate at which displacement changes with time.
- $v = \frac{ds}{dt}$ (Gradient of distance–time graph is velocity)
- Acceleration is the rate at which velocity changes with time.
- $a = \frac{dv}{dt}$ (Gradient of velocity–time graph is acceleration)
- displacement *differentiate* velocity *differentiate* acceleration
 (s) $\rightarrow$ $(v = \frac{ds}{dt})$ $\rightarrow$ $(a = \frac{dv}{dt})$

Example 2

A pebble is thrown vertically upwards such that its height, s m, above the top of the cliff after t s is given by

$$s = 10t - 5t^2$$

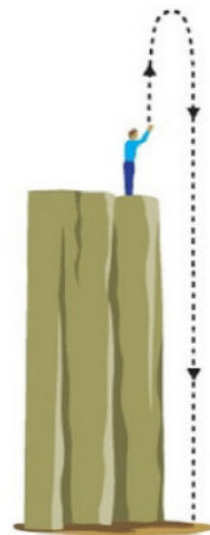
Find the maximum height reached by the pebble and when this occurs.

$$s = 10t - 5t^2$$

$$v = \frac{ds}{dt} = 10 - 10t$$

At the maximum height the velocity is zero $\Rightarrow 10 - 10t = 0 \Rightarrow t = 1$

$$\text{At } t = 1 \text{ s} \quad s_{\max} = 10(1) - 5(1)^2 \quad s_{\max} = 5 \text{ m}$$



Questions on Kinematics

- 1 ►** The displacement, s m, of a particle after t s from a fixed point O is given by
 $s = 10t^2 - 30t + 1$
After a time of 2 s find the particle's
a velocity in m/s **b** acceleration in m/s^2
- 2 ►** The displacement, s m, of a particle after t s from a fixed point O is given by
 $s = 10 + 7t - t^2$
After a time of 3 s find the particle's
a velocity in m/s **b** acceleration in m/s^2
- 3 ►** The displacement, s m, of a particle after t s from a fixed point O is given by
 $s = t^3 + 2t^2 - 3t + 1$
After a time of 2 s find the particle's
a velocity in m/s **b** acceleration in m/s^2
- 4 ►** The displacement, s m, of a particle after t s from a fixed point O is given by
 $s = 20 + 12t + 3t^2 - t^3$
After a time of 2 s find the particle's
a velocity in m/s **b** acceleration in m/s^2
- 5 ►** A tennis ball is projected vertically upwards such that its height, s m, after t s is given by
 $s = 16t - 4t^2$
Find the maximum height reached by the ball and when this occurs.
- 6 ►** The velocity of a stone, v m/s, t s after it is thrown upwards is given by
 $v = 4 + 12t - t^2$
Calculate the stone's
a velocity after 2 s
b acceleration after 2 s
c maximum velocity.

Q6c HINT

Maximum velocity
occurs when
acceleration is
zero.

Solutions

1 ► a $v = 20t - 30 \text{ m/s}$, 10 m/s

b $a = 20 \text{ m/s}^2$

2 ► a $v = 7 - 2t \text{ m/s}$, 1 m/s

b $a = -2 \text{ m/s}^2$

3 ► a $v = 3t^2 + 4t - 3 \text{ m/s}$, 17 m/s

b $a = 6t + 4 \text{ m/s}^2$, 16 m/s^2

4 ► a $v = 12 + 6t - 3t^2 \text{ m/s}$, 12 m/s

b $a = 6 - 6t \text{ m/s}^2$, -6 m/s^2

5 ► $v = 16 - 8t$, $v = 0$ at $t = 2$, $s_{\text{max}} = 16 \text{ m}$

6 ► a $v = 24 \text{ m/s}$

b $a = 12 - 2t \text{ m/s}^2$, $a = 8 \text{ m/s}^2$

c $t = 6$, $v_{\text{max}} = 40 \text{ m/s}$