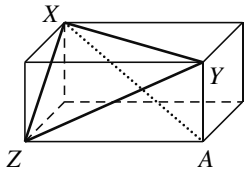
14. The diagram shows a cuboid. In triangle XYZ, the lengths of XY, XZ and YZ are 9, 8 and $\sqrt{55}$ respectively.

What is the length of the diagonal XA shown?



15. The equation $x^2 - bx + 80 = 0$, where $b > 0$, has two integer-valued solutions. What is the sum of the possible values of b?

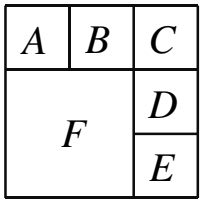
16. Given that $a + b = 5$ and $ab = 3$, what is the value of $a^4 + b^4$?

17. David removed one number from ten consecutive natural numbers. The sum of the remaining numbers was 2012.

Which number did he remove?

18. The diagram shows a square divided into six smaller squares labelled A, B, C, D, E and F. Two squares are considered to be adjacent if they have more than one point in common. The numbers 1, 2, 3, 4, 5 and 6 are to be placed in the smaller squares, one in each, so that no two adjacent squares contain numbers differing by 3.

How many different arrangements are possible?



19. A rectangle which has integer-length sides and area 36 is cut from a square with sides of length 20 so that one of the sides of the rectangle forms part of one of the sides of the square.

What is the largest possible perimeter of the remaining shape?

20. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ exist in which the sum of the largest element and the smallest element is 11?

SENIOR 'KANGAROO' MATHEMATICAL CHALLENGE

Friday 30th November 2012

Organised by the United Kingdom Mathematics Trust

The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.

   Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.
5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

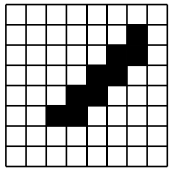
Enquiries about the Senior Kangaroo should be sent to:

Maths Challenges Office, School of Maths Satellite,
University of Leeds, Leeds, LS2 9JT
Tel. 0113 343 2339
www.ukmt.org.uk

1. How many zeroes are there at the end of the number which is the product of the first 2012 prime numbers?

2. The size of the increase from each term to the next in the list a, $225\frac{1}{2}$, c, d, 284 is always the same.
What is the value of a?

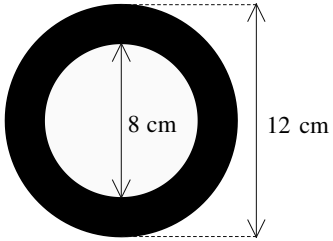
3. On the grid shown in the diagram, the shaded squares form a region, A.



What is the maximum number of additional grid squares which can be shaded to form a region B such that B contains A and that the lengths of the perimeters of A and B are the same?

4. Five cards are laid on a table, as shown. Every card has a letter on one side and a number on the other side.
Peter says: "For every card on the table, if there is a vowel on one side of the card, then there is an even number on the other side."
What is the smallest number of cards Sylvia must turn over in order to be certain that Peter is telling the truth?

5. Susan has two pendants made of the same material. They are equally thick and weigh the same. The first pendant is in the shape of an annulus created from two concentric circles, with diameters 8 cm and 12 cm, as shown. The shape of the second pendant is a disc. The diameter of the second pendant is written in the form $a\sqrt{b}$, where a is an integer and b is a prime integer.



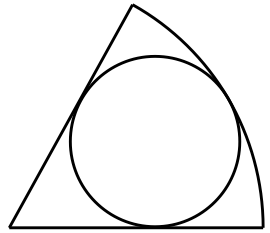
What is the value of $a + b$?

6. Given that $4^x = 9$ and $9^y = 256$, what is the value of xy?

7. When 1001 is divided by a single-digit number, the remainder is 5.
What is the remainder when 2012 is divided by the same single-digit number?

8. The three prime numbers a, b and c are such that $a > b > c$, $a + b + c = 52$ and $a - b - c = 22$.
What is the value of abc?

9. The diagram shows a circle touching a sector of another circle in three places. The ratio of the radius of the sector to the radius of the small circle is 3:1. The ratio of the area of the sector to the area of the small circle, written in its simplest form, is $p : q$.



What is the value of $p + q$?

10. Sixteen teams play in a volleyball league. Each team plays one game against every other team. For each game, the winning team is awarded 1 point, and the losing team 0 points. There are no draws. After all the games have been played and the teams have been ranked according to their total scores, the total scores form a sequence where the difference between consecutive terms is constant.
How many points did the team in first place receive?

11. Last year there were 30 more boys than girls in the school choir. This year the number of choir members has increased by 10%, the number of girls has increased by 20% and the number of boys by 5%.
How many members does the choir have this year?

12. The cells of a 4×4 grid are coloured black and white as shown in Figure 1. One move allows us to exchange the colourings of any two cells positioned in the same row or in the same column.
What is the minimum number of moves needed to obtain Figure 2?

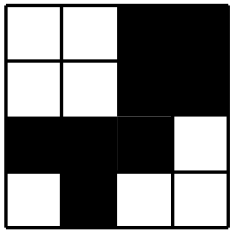


Figure 1

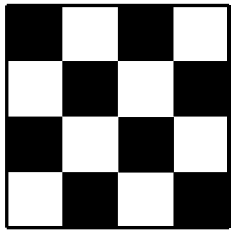
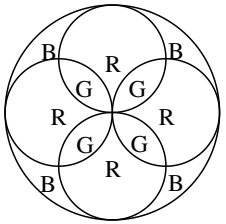


Figure 2

13. A circular stained-glass window is shown in the diagram. The four smaller circles are the same size and are positioned at equal intervals around the centre of the large circle. The letters R, G and B have been placed in regions of red, green and blue glass respectively. The total area of the green glass is 400.



What is the area of the blue glass?