

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
January 2012

Mathematics

MPC3

Unit Pure Core 3

Friday 20 January 2012 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



J A N 1 2 M P C 3 0 1

Answer **all** questions in the spaces provided.

1 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for $\int_0^3 4^x dx$.
(4 marks)

(b) A curve is defined by the equation $y = 4^x$. The curve intersects the line $y = 8 - 2x$ at a single point where $x = \alpha$.

(i) Show that α lies between 1.2 and 1.3. (2 marks)

(ii) The equation $4^x = 8 - 2x$ can be rearranged into the form $x = \frac{\ln(8 - 2x)}{\ln 4}$.

Use the iterative formula $x_{n+1} = \frac{\ln(8 - 2x_n)}{\ln 4}$ with $x_1 = 1.2$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

QUESTION
PART
REFERENCE



QUESTION
PART
REFERENCE

Turn over ►



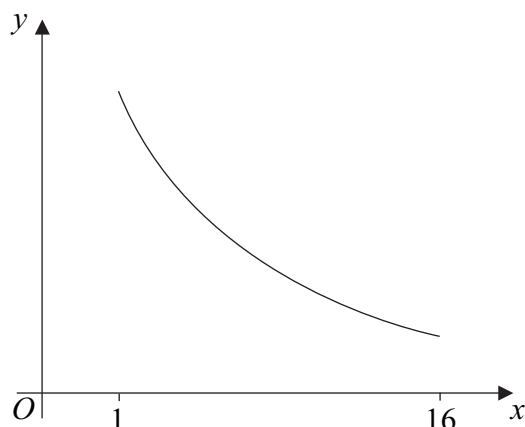
[illegible]

QUESTION
PART
REFERENCE

Turn over ►



- 2** The curve with equation $y = \frac{63}{4x-1}$ is sketched below for $1 \leq x \leq 16$.



The function f is defined by $f(x) = \frac{63}{4x-1}$ for $1 \leq x \leq 16$.

- (a) Find the range of f . (2 marks)
- (b) The inverse of f is f^{-1} .
- (i) Find $f^{-1}(x)$. (3 marks)
- (ii) Solve the equation $f^{-1}(x) = 1$. (2 marks)
- (c) The function g is defined by $g(x) = x^2$ for $-4 \leq x \leq -1$.
- (i) Write down an expression for $fg(x)$. (1 mark)
- (ii) Solve the equation $fg(x) = 1$. (3 marks)

QUESTION
PART
REFERENCE



QUESTION
PART
REFERENCE

Turn over ►



QUESTION
PART
REFERENCE

This image shows a full page of primary-ruled paper. It features a vertical solid line on the left side, creating a narrow margin. The rest of the page is filled with horizontal dashed lines, providing a guide for letter height and placement. There are no markings or text on the page.



3 (a) Given that $y = 4x^3 - 6x + 1$, find $\frac{dy}{dx}$. (1 mark)

(b) Hence find $\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1} dx$, giving your answer in the form $p \ln q$, where p and q are rational numbers. (5 marks)

QUESTION
PART
REFERENCE



[illegible]

4 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2 \theta = 3(3 - \sec \theta)$$

giving all solutions to the nearest 0.1° in the interval $0^\circ < \theta < 360^\circ$. (6 marks)

(b) Hence solve the equation

$$\tan^2(4x - 10^\circ) = 3[3 - \sec(4x - 10^\circ)]$$

giving all solutions to the nearest 0.1° in the interval $0^\circ < x < 90^\circ$. (3 marks)

QUESTION
PART
REFERENCE

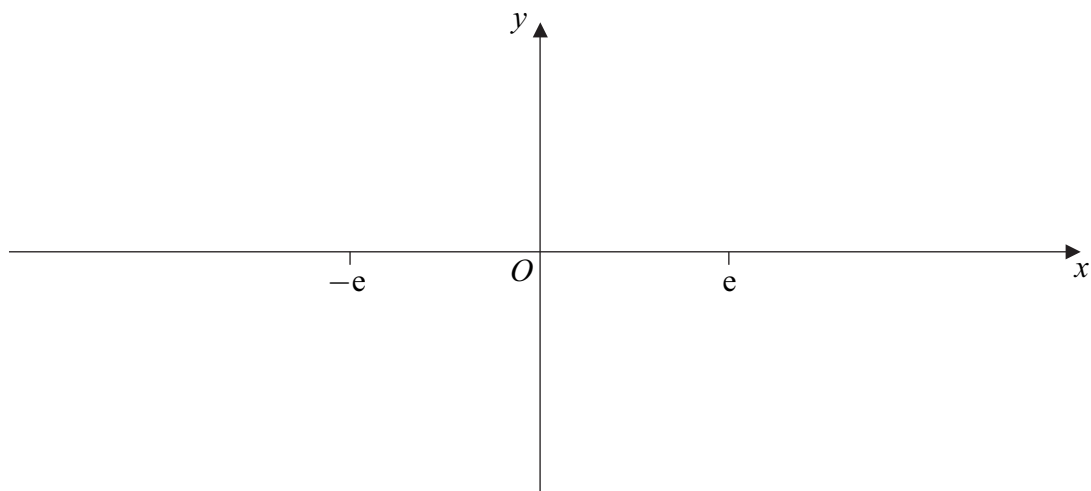


[illegible]

- 5 (a)** Describe a sequence of two geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4 \ln(x - e)$. (4 marks)
- (b)** Sketch, on the axes given below, the graph of $y = |4 \ln(x - e)|$, indicating the exact value of the x -coordinate where the curve meets the x -axis. (3 marks)
- (c) (i)** Solve the equation $|4 \ln(x - e)| = 4$. (3 marks)
- (ii)** Hence, or otherwise, solve the inequality $|4 \ln(x - e)| \geq 4$. (3 marks)

QUESTION
PART
REFERENCE

(b)



[illegible]

6 (a) Given that $x = \frac{1}{\sin \theta}$, use the quotient rule to show that $\frac{dx}{d\theta} = -\operatorname{cosec} \theta \cot \theta$.
(3 marks)

(b) Use the substitution $x = \operatorname{cosec} \theta$ to find $\int_{\sqrt{2}}^2 \frac{1}{x^2 \sqrt{x^2 - 1}} dx$, giving your answer to three significant figures.
(9 marks)

QUESTION
PART
REFERENCE



[illegible]

7 (a) A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates. (7 marks)

(b) (i) Use integration by parts twice to find the exact value of $\int_0^4 x^2 e^{-\frac{x}{4}} dx$. (7 marks)

(ii) The region bounded by the curve $y = 3xe^{-\frac{x}{8}}$, the x -axis from 0 to 4 and the line $x = 4$ is rotated through 360° about the x -axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated. (2 marks)

QUESTION
PART
REFERENCE



[illegible]

[illegible]

[illegible]

There are no questions printed on this page

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



There are no questions printed on this page

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**



There are no questions printed on this page

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**

