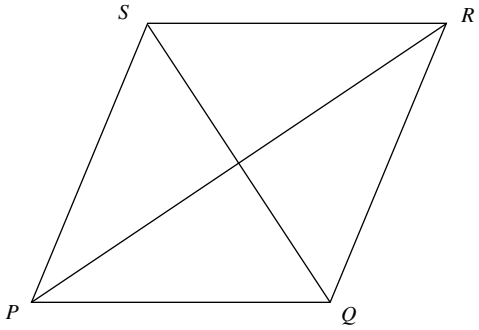
13. Zoe was born on her mother's 24th birthday so they share birthdays. Assuming they both live long lives, on how many birthdays will Zoe's age be a factor of her mother's age?

14. What is the largest three-digit integer that can be written in the form $n + \sqrt{n}$ where n is an integer?

15. How many integers a are there for which the roots of the quadratic equation $x^2 + ax + 2013 = 0$ are integers?

16. A sphere of radius 3 has its centre at the origin. How many points on the surface of the sphere have coordinates that are all integers?

17. The length of each side of the rhombus $PQRS$ is equal to the geometric mean of the lengths of its diagonals. What is the size in degrees of the obtuse angle PQR?



[The geometric mean of 2 values x_1 and x_2 is given by $\sqrt{x_1 x_2}$.]

18. How many of the first 2013 triangular numbers are multiples of 5?

19. The increasing sequence 1, 3, 4, 9, 10, 12, 13, … contains all the powers of 3 and all the numbers that can be written as the sum of two or more distinct powers of 3. What is the 70th number in the sequence?

20. Rachel and Nicky stand at either end of a straight track. They then run at constant (but different) speeds to the other end of the track, turn and run back to their original end at the same speed they ran before. On their first leg, they pass each other 20 m from one end of the track. When they are both on their return leg, they pass each other for a second time 10 m from the other end of the track. How many metres long is the track?

SENIOR 'KANGAROO' MATHEMATICAL CHALLENGE

Friday 29th November 2013

Organised by the United Kingdom Mathematics Trust

The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.

   Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.
5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

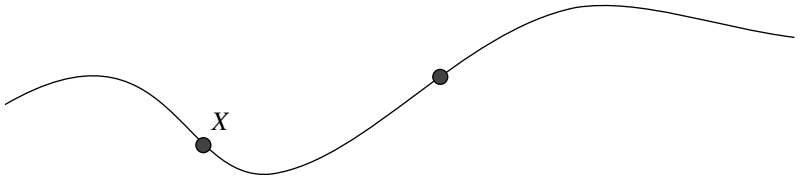
Enquiries about the Senior Kangaroo should be sent to:

Maths Challenges Office, School of Maths Satellite,
University of Leeds, Leeds, LS2 9JT
Tel. 0113 343 2339
www.ukmt.org.uk

1. Adam, Bill and Carl have 30 sweets between them. Bill gives 5 sweets to Carl, Carl gives 4 sweets to Adam and Adam gives 2 sweets to Bill. Now each of them has the same number of sweets. How many sweets did Carl have initially?

2. An i-rectangle is defined to be a rectangle all of whose sides have integer length. Two i-rectangles are considered to be the same if they have the same side-lengths. The sum of the areas of all the different i-rectangles with perimeter 22 cm is A cm^2. What is the value of A?

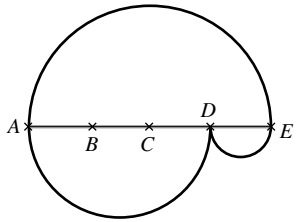
3. Some historians claim that the ancient Egyptians used a rope with two knots tied in it to construct a right-angled triangle by joining the two ends of the rope and taking the vertices of the triangle to be at the two knots and at the join. The length of the rope shown is 60 m and one of the knots is at X, which is 15 m from one end of the rope. How many metres from the other end of the rope should the second knot be placed to be able to create a right-angled triangle with the right angle at X?



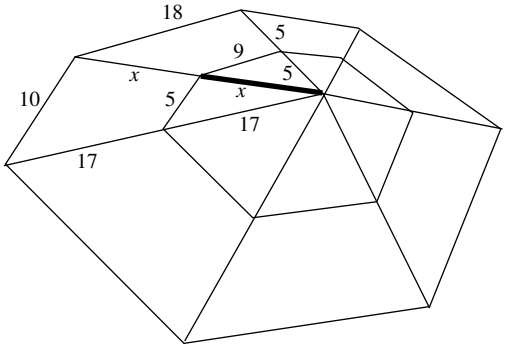
4. The height, width and length of a cube are multiplied by 2, 3 and 6 respectively to create a cuboid. The surface area of the cuboid is N times the surface area of the original cube. What is the value of N?

5. In a university admissions test, Dean gets exactly 10 of the first 15 questions correct. He then answers all the remaining questions correctly. Dean finds out he has answered 80% of all the questions correctly. How many questions are there on the test?

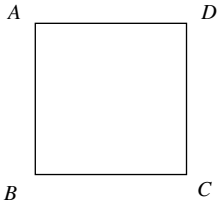
6. In the diagram, AE is divided into four equal parts and semicircles have been drawn with AE, AD and DE as diameters. This has created two new paths, an upper path and a lower path, from A to E. The ratio of the length of the upper path to the length of the lower path can be written as $a : b$ in its lowest terms. What is the value of $a + b$?



7. A mathematically skilful spider has spun a web and the lengths of some of the strands (which are all straight lines) are as shown in the diagram. It is known that x is an integer. What is the value of x?



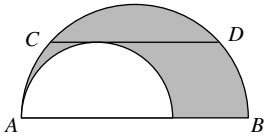
8. The square $ABCD$ has sides of length 1. All possible squares that share two vertices with $ABCD$ are drawn. The boundary of the region formed by the union of these squares is an irregular polygon. What is the area of this polygon?



9. In triangle ABC, angle B is 25% smaller than angle C and 50% larger than angle A. What is the size in degrees of angle B?

10. In the equation $2^{m+1} + 2^m = 3^{n+2} - 3^n$, m and n are integers. What is the value of m?

11. The diagram shows two semicircles. The chord CD of the larger semicircle is parallel to AB, and touches the smaller semicircle. The length of CD is 32 m. The area of the shaded region is $k\pi$ m^2. What is the value of k?



12. The sum of five consecutive integers is equal to the sum of the next three consecutive integers. What is the largest of these eight integers?