UK Junior Mathematical Olympiad 2018

Organised by The United Kingdom Mathematics Trust

Tuesday 12th June 2018

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. **Write in blue or black pen or pencil.**
   For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.
   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.
   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 30 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

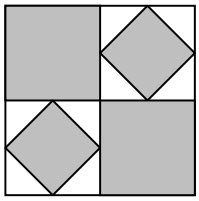
Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the value of 0.8×0.12?

A2. A large square is split into four congruent squares, two of which are shaded. The other two squares have smaller shaded squares drawn in them whose vertices are the midpoints of the sides of the unshaded squares.

What fraction of the large square is shaded?



A3. What is the largest integer for which each pair of consecutive digits is a square?

A4. What is the value of $\frac{10^5}{5^5}$?

A5. The sizes in degrees of the interior angles of a pentagon are consecutive even numbers.

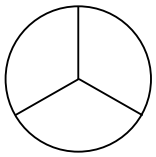
What is the size of the largest of these angles?

A6. A two-digit number '*ab*' is multiplied by its reverse '*ba*'. The ones (units) and tens digits of the four-digit answer are both 0.

What is the value of the smallest such two-digit number '*ab*'?

A7. The diagram shows a circle divided into three equal sectors.

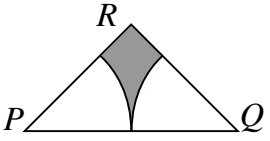
What is the ratio of the length of the perimeter of one of these sectors to the length of the circumference of the circle?



A8. How many three-digit integers less than 1000 have exactly two different digits in their representation (for example, 232, or 466)?

A9. The triangle PQR is isosceles with $PR = QR$. Angle $PRQ = 90°$ and length $PQ = 2$ cm. Two arcs of radius 1 cm are drawn inside triangle PQR. One arc has its centre at P and intersects PR and PQ. The other arc has its centre at Q and intersects QR and PQ.

What is the area of the shaded region, in cm^2 ?



A10. A four-digit integer has its digits increasing from left to right. When we reverse the order of the digits, we obtain a four-digit integer whose digits decrease from left to right. A third four-digit integer uses exactly the same digits, but in a different order. The sum of the three integers is 26 352.

What is the value of the smallest of the three integers?

Section B

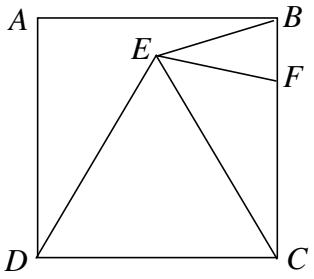
Your solutions to Section B will have a major effect on your JMO result. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1. Polly Garter had her first child on her 20th birthday, her second child exactly two years later, and her third child exactly two years after that.

How old was Polly when her age was equal to the sum of her three children's ages?

B2. In the diagram shown, $ABCD$ is a square and point F lies on BC. Triangle DEC is equilateral and $EB = EF$.

What is the size of $\angle CEF$?



B3. The letters a, b and c stand for non-zero digits. The integer 'abc' is a multiple of 3; the integer '$cbabc$' is a multiple of 15; and the integer '$abcba$' is a multiple of 8.

What is the integer 'abc'?

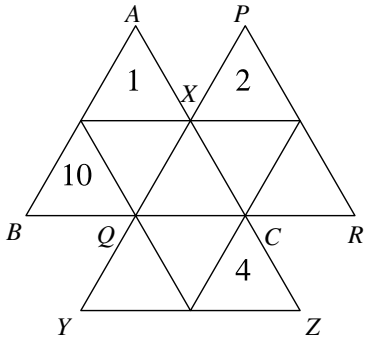
B4. A rectangular sheet of paper is labelled $ABCD$, with AB one of the longer sides. The sheet is folded so that vertex A is placed exactly on top of the opposite vertex C. The fold line is XY, where X lies on AB and Y lies on CD.

Prove that triangle CXY is isosceles.

B5. The diagram shows three triangles, ABC, PQR and XYZ, each of which is divided up into four smaller triangles. The diagram is to be completed so that the positive integers from 1 to 10 inclusive are placed, one per small triangle, in the ten small triangles. The totals of the numbers in the three triangles ABC, PQR and XYZ are the same.

Numbers 1, 2, 4 and 10 have already been placed.

In how many different ways can the diagram be completed?



B6. Sixteen counters, which are black on one side and white on the other, are arranged in a 4 by 4 square. Initially all the counters are facing black side up. In one 'move', you must choose a 2 by 2 square within the square and turn all four counters over once. Describe a sequence of 'moves' of minimum length that finishes with the colours of the counters of the 4 by 4 square alternating (as shown in the diagram).