AS

Mathematics

MPC1 – Pure Core 1

Mark scheme

6360

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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$y = \pm\frac{5}{3}x + \ldots$	M1		
				$y = -\frac{5}{3}x - 1$ for guidance
	$m = -\frac{5}{3}$	A1	2	**must** see $m = \ldots$ or statement such as "AB has gradient $-\frac{5}{3}$ so line parallel to AB also has gradient $-\frac{5}{3}$."
(b)	$5x+3y+3=0$ & $3x-2y+17=0$ eg $10x+6+9x+51=0$	M1		**correct** equations used and **correct** elimination of x or y eg $19x+57=0$ or $19y-76=0$ etc
	$x=-3$ **or** $x=-\frac{57}{19}$ **or** $y=4$ **or** $y=\frac{76}{19}$	A1		either x or y correct in any equivalent form
	{both $x=-3$ **and** $y=4$ **}** **or** $(-3,4)$	A1	3	both coordinates written as integers
(c)	$5(2k+3)+3(4-3k)+3=0$ $10k+15 \quad + \quad 12-9k \quad +3=0$	M1		correct substitution into correct equation & correct expansion of brackets
	$k=-30$	A1	2	
	Total		**7**	

(a) Do not penalise incorrect rearrangement if $m = -\frac{5}{3}$ is stated.

Example $y=-\frac{5}{3}x-3$ so $m = -\frac{5}{3}$ scores **M1 A1**

NMS $m = -\frac{5}{3}$ earns 2 marks. **NMS** $-\frac{5}{3}$ earns **M1 A0**. **NMS** $(m =)\ \frac{5}{3}$ earns **M1 A0**.

NMS Award **M1 A0** only for "$m = -\frac{5}{3}x$".

(b) $5\left(\frac{2y}{3}-\frac{17}{3}\right)+3y+3=0$ earns **M1,** however $5\left(\frac{2y}{3}+\frac{17}{3}\right)+3y+3=0$, for example, scores **M0.**

Other examples scoring **M1** are $3x-2\left(-\frac{5}{3}x-1\right)+17=0$; $-\frac{5}{3}x-1=\frac{3}{2}x+\frac{17}{2}$

Accept any correct equivalent fraction for first **A1** but must have **both** $x=-3$ **and** $y=4$ for final **A1.**

NMS $(-3,4)$ scores **3 marks**

Q2	Solution	Mark	Total	Comment
(a)	45	B1	1	
(b)	$\dfrac{**+\sqrt{5}}{7+3\sqrt{5}} \times \dfrac{7-3\sqrt{5}}{7-3\sqrt{5}}$	M1		
	(Numerator =) $315+7\sqrt{5}-135\sqrt{5}-15$	A1		at least this far
	(Denominator = $49+21\sqrt{5}-21\sqrt{5}-45$) $= 4$	B1		must be seen as denominator
	Value $= \dfrac{300-128\sqrt{5}}{4}$ $= 75-32\sqrt{5}$	A1cso	4	
	Total		5	

NO MISREADS ALLOWED IN THIS QUESTION

(b) Condone multiplication by $7-3\sqrt{5}$ instead of $\times\dfrac{7-3\sqrt{5}}{7-3\sqrt{5}}$ for **M1 only if** subsequent working shows multiplication by both numerator and denominator – otherwise **M0**

For first **A1** 45×7 , 45×3 and 3×5 must be evaluated correctly

An error in the denominator such as $49+7\sqrt{5}-7\sqrt{5}-45=4$ should be given **B0** and it would then automatically lose the final **A1cso**

May use alternative conjugate $\times\dfrac{3\sqrt{5}-7}{3\sqrt{5}-7}$ **M1 ;** numerator = $-315-7\sqrt{5}+135\sqrt{5}+15$ **A1** etc

Q3	Solution	Mark	Total	Comment
(a)(i)	$\left(x-\frac{7}{2}\right)^2 \ldots$	**M1**		$(x-3.5)^2$ **… OE**
	$\left(x-\frac{7}{2}\right)^2-\frac{41}{4}$	**A1**	**2**	$(x-3.5)^2-10.25$
(ii)	(Minimum value =) -10.25 **OE**	**B1F**	**1**	**must FT** their q
(b)	Translation	**E1**		or translate(d) (by/through) (*and no other transformation given*)
	$\begin{bmatrix} 0.5 \\ * \end{bmatrix}$	**M1**		
	$\begin{bmatrix} 0.5 \\ 10.25 \end{bmatrix}$	**A1**	**3**	must express as vector to earn **A1** mark
	Total		**6**	
(a)(i)	If **M1** is not earned, award **SC1** for $\left(x-\frac{7}{2}\right)-\frac{41}{4}$			
(ii)	Do **NOT** accept any **pair** of values. **Example** (3.5, –10.25) scores **B0** since this is hedging bets Condone $y=\text{"}their\text{"}\,q$ for **B1** but $x=\text{"}their\text{"}\,q$ scores **B0**			
(b)	Do **NOT** accept "shift", "move", "slide", "transformation", "trans" etc for **E1** Accept "0.5 in x-direction" , " $\frac{1}{2}$ to the right" , "(0.5,*)" for **M1 only**			

Q4	Solution	Mark	Total	Comment
(a)(i)	$(\text{p}(-3)) = (-3)^3 - 5(-3)^2 - 8(-3) + 48$	M1		clear attempt at p(–3) NOT long division
	$= -27 - 45 + 24 + 48$ $= 0$ therefore $x+3$ is a factor	A1	2	must see powers of –3 simplified correctly working showing that p(–3)=0 and correct statement
(ii)	$x^2 + bx + c$ *with* $b = -8$ **or** $c = 16$	M1		by inspection
	$x^2 - 8x + 16$	A1		may see as quotient in long division
	$(\text{p}(x) =)\ (x+3)(x-4)(x-4)$	A1	3	must see product
(b)(i)	$\text{p}(2) = 2^3 - 5\times2^2 - 8\times2 + 48$	M1		clear attempt at p(2) NOT long division
	= 8–20–16+48 (Remainder =) 20	A1	2	
(ii)	Quadratic factor $x^2 + bx + c$			
	$b = -3$ **or** $c = -14$	M1		by inspection
	$x^2 - 3x - 14$	A1		may see as quotient in long division
	$(\text{p}(x) =)\ (x-2)(x^2 - 3x - 14) + 20$	A1	3	must see full correct expression
	Total		**10**	

(a)(i) Minimum required for statement is "$\therefore$ factor"
Powers of –3 must be evaluated: **Example "**p(–3) $= -27 - 45 + 24 + 48$ = 0 so factor" scores **M1 A1**
Statement may appear first :
Example " x+3 is factor if p(–3) = 0 & p(–3) $= -27 - 45 + 24 + 48 = 0$" scores **M1 A1**
However, **Example "** $\text{p}(-3) = (-3)^3 - 5(-3)^2 - 8(-3) + 48 = 0$ therefore x+3 is a factor" scores **M1 A0**

(ii) **M1** may also be earned for a full long division attempt by (x+3), or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .
M1 may also be earned for ***showing*** p(4) =0 and ***stating*** that $(x-4)$ is a factor
NMS $\text{p}(x) = (x+3)(x-4)^2$ scores **3 marks;**

(b)(i) Do not apply ISW for eg " p(2) = 20, therefore remainder is – 20"
May use "their" product of factors $\text{p}(2) = (2+3)(2-4)(2-4)$ for **M1** and **A1** if factors and working are all correct giving 20.

(ii) **M1** may be earned for a full long division attempt by (x–2), or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .
M1 may also be earned for using their value from part **(b)(i)** for r and a full attempt to find b and c.

Q5	Solution	Mark	Total	Comment
(a)	$(x-5)^2+(y+3)^2$ =…	M1		**or** $(x-5)^2+(y--3)^2$ =…
	7^2+4^2 **or** $49+16$ **or** 65	B1		**or** seen under square root
	$(x-5)^2+(y+3)^2=65$	A1	3	**or** $(x-5)^2+(y--3)^2=65$
(b)	$x_B=12$	B1		
	$y_B=-7$	B1	2	$B(12,-7)$
(c)	Grad AC = $\dfrac{1--3}{-2-5}$	M1		condone one sign error in one term FT their B if grad AB or grad BC is used.
	= $-\dfrac{4}{7}$	A1		
	Grad tgt $=\dfrac{7}{4}$	B1F		
	Equation of tgt: $y-1="their"\dfrac{7}{4}(x--2)$	m1		or $y="their"\dfrac{7}{4}x+c$ & attempt to find c using $x=-2$ and $y=1$
	$7x-4y+18=0$	A1	5	any multiple – must have integer coefficients and all terms on one side
(d)	$CT^2=AT^2+AC^2$			
	$(CT^2=)\ 4^2+"their"65$	M1		Pythagoras with hyp=CT & AC^2=”*their*” k or correct
	$(CT^2=)\ 81$	A1		or $(CT=)\sqrt{81}$
	$(CT=)9$	A1	3	all notation correct; must simplify $\sqrt{81}$
	Total		**13**	

(a) **NMS** $(x-5)^2+(y+3)^2=65$ scores **3 marks**

allow RHS = $\left(\sqrt{65}\right)^2$ instead of 65 for full marks

Example: $(x-5)^2+(y+3)^2=\sqrt{65}$ earns **M1 B1 A0**

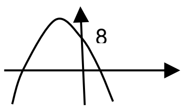
Equation of circle must be written explicitly as $(x-5)^2+(y+3)^2=65$ to earn **A1** mark

(c) Award **M1 A0** for grad AC =4/7

For **m1** candidate must be attempting equation of tangent; if **B1F** not earned and their gradient of AC is m then award **m1** if using 1/*m* or –*m* and correct coordinates (–2,1).

For final **A1** accept answers such as $0=8y-14x-36$ but **NOT** $7x-4y=-18$

(d) **Example:** $4^2+65=81=9$ scores **M1, A1, A0 ; Example:** $4^2+65=81,\ \sqrt{81}=9$ scores **M1, A1, A1**

Q6	Solution	Mark	Total	Comment
(a)(i)	$(x=)\ \dfrac{4\pm\sqrt{80}}{-4}$, or $(x=)\ \dfrac{-4\pm\sqrt{80}}{4}$ or $(x=)\ \dfrac{-2\pm\sqrt{20}}{2}$	**M1**		if completing square must have at least $x+1=\pm\sqrt{5}$
	$(x=)\ -1\pm\sqrt{5}$	**A1**	**2**	do **not** accept $-1\pm-\sqrt{5}$ for **A1**
(ii)		**M1** **A1**	**2**	$\wedge$ shape as shown in all 4 quadrants , max to left of y-axis with y-intercept 8 stated/marked
(b)(i)	$k(x+4)=8-4x-2x^2$			
	$2x^2+kx+4x+4k-8=0$			must expand $k(x+4)$ & have all terms on one side with =0 before final line
	$2x^2+(k+4)x+4(k-2)=0$	**B1**	**1**	**AG** be convinced
(ii)	$(k+4)^2-4\times2\times4(k-2)\quad(=0)$	**M1**		correct discriminant
	$k^2-24k+80\quad(=0)$	**A1**		
	$k=4,\quad k=20$	**A1cso**	**3**	
	Total		**8**	

(a)(ii) Withhold **A1** if maximum y- value is clearly not greater than 8, or graph has wrong curvature in third and fourth quadrants.
Do not withhold **A1** if incorrect x-intercepts are marked on x-axis, etc.
Accept (0,8) stated or marked on y-axis as y-intercept, but do NOT accept (8,0).

(b)(i) Must have "=0" on final line but this may be on LHS.
Do not accept incorrect "trailing equal" signs, ie from line 1 to line 2 of proof.

(ii) Condone poor use/omission of brackets for **M1** if correct discriminant is intended, but the **A1 cso** cannot then be earned even if recovered later. Candidates must have "= 0" on at least one line of working or statement "$b^2-4ac=0$" and all working correct to earn **A1cso.**
If candidate uses " > 0" etc then withhold **A1cso** even if final answer is written as $k=4,\quad k=20$**.**

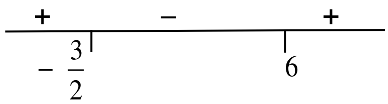
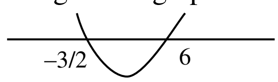
Q7	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{dy}{dx}=\right)\quad -2x-9x^2$	M1 A1		one term correct all correct (no +c etc)
	when $x=-2$, $\frac{dy}{dx}=(4-36=)-32$	A1		
	$y=\text{"}their-32\text{"}x+c$ & attempt to find c using $x=-2$ and $y=24$	m1		or $y-24=\text{"}their-32\text{"}(x--2)$
	$y=-32x-40$	A1	5	must write in this form; no **ISW** here
(ii)	$y=0\Rightarrow x=-\frac{5}{4}$ **OE**	B1F	1	**strict FT** from their answer to **(a)(i)**
(b)(i)	$4x-\frac{x^3}{3}-\frac{3x^4}{4}\ (+c)$	M1 A1		two terms correct all correct
	$\left[4\times1-\frac{1^3}{3}-\frac{3\times1^4}{4}\right]-\left[4\times(-2)-\frac{(-2)^3}{3}-\frac{3(-2)^4}{4}\right]$	m1		"their" F(1) – F(–2)
	$\left[4-\frac{1}{3}-\frac{3}{4}\right]-\left[-8+\frac{8}{3}-\frac{48}{4}\right]$	A1		**correct** with powers of 1 and (–2) and minus signs handled correctly
	$=20\frac{1}{4}$	A1	5	20.25 , $\frac{81}{4}$, $\frac{243}{12}$ **OE**
(ii)	Area of missing triangle $=\left(\frac{1}{2}\times24\times\frac{3}{4}=\right)9$	B1		or correct **single** equivalent fraction
	Area of region = "their"**(b)(i)** –"their" Δ	M1		"their" ($20\frac{1}{4}$–9)
	$=11\frac{1}{4}$	A1	3	11.25 , $\frac{45}{4}$, $\frac{135}{12}$ **OE**
	Total		**14**	

(a)(i) Must see $y=-32x-40$ explicitly for final **A1**; ie not enough to see $y=-32x+c$ with $c=-40$ appearing on later line.

(a)(ii) Allow $-\frac{40}{32}$ etc.

(b)(i) Must combine terms for final **A1; Example** $\ldots 3\frac{1}{4}+17$ scores final **A0.**

(ii) May find triangle area by considering trapezium with one side of zero length or integration for **B1**.
For **M1** condone use of "their" Δ –"their"**(b)(i)** if appropriate for their values.
Be generous in awarding this **M1** provided you are convinced they are considering the area of a triangle.

Q8	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\right) 27-12x$	**M1** **A1**	**2**	one term correct all correct (no $+c$ etc)
(ii)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) 54+27\times\left(-\frac{3}{2}\right)-6\times\left(-\frac{3}{2}\right)^2$	**M1**		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=54-\frac{81}{2}-\frac{54}{4}=0$	**A1**		convincingly showing $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ **and** $\frac{\mathrm{d}y}{\mathrm{d}x}=...$ **must** appear on at least one line
	$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\right) 27-12\times\left(\frac{-3}{2}\right)$	**M1**		correct substitution into "their" $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=27+18\ (=45) > 0$ $\Rightarrow P$ is minimum point	**A1cso**	**4**	correct working and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ used and value shown to be > 0 with correct statement(s) must earn **3** previous marks to earn **A1cso**
(b)(i)	(Decreasing so) $54+27x-6x^2<0$	**M1**		
	$6x^2-27x-54>0$ $2x^2-9x-18>0$	**A1**	**2**	**AG** be convinced
(ii)	$(2x+3)(x-6)$	**M1**		correct factors or correct use of formula as far as $\frac{9\pm\sqrt{225}}{4}$
	CVs are $x=-\frac{3}{2},\ x=6$	**A1**		condone equivalent fractions here
		**M1**		use of sign diagram or graph 
	$x<-\frac{3}{2},\quad x>6$	**A1**	**4**	fractions must be simplified for final mark no **ISW** here
	Total		**12**	

(b)(ii) For second **M1**, if critical values are correct then sign diagram or sketch must be correct ***with correct CVs marked***.
However, if CVs are not correct then second **M1** can be earned for attempt at sketch or sign diagram but ***their CVs*** MUST be marked on the diagram or sketch.
Final **A1,** inequality must have x and no other letter.

Final answer of $x<-\frac{3}{2}$ OR $x>6$ (with or without working) scores **4 marks**.

(A) $k<-\frac{3}{2},\quad k>6$ (B) $x<-\frac{3}{2}$ AND $x>6$ (C) $x\le-\frac{3}{2},\quad x\ge6$

with or without working, each score **3 marks** (**SC3**)

Example NMS $x<\frac{3}{2},\quad x>6$ scores **M0** (since one CV is incorrect)

Example NMS $x>-1.5,\quad x>6$ scores **M1 A1 M0** (since both CVs are correct)