AQA

AS
Mathematics

MPC2 Pure Core 2
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	Perimeter of sector = $8+8+8\theta$	M1		$r+r+r\theta$ used for the perimeter PI by eg $8\theta = 22-16$ OE
	$\theta = 0.75$	A1	2	A correct value for θ. eg 6/8 NMS scores 2/2
(b)	(Area of sector) = $\frac{1}{2}r^2\theta = \frac{1}{2}(8^2)\theta$	M1		$\frac{1}{2}r^2\theta$ seen, or used, for the sector area OE eg $\frac{1}{2}rL$ with $L = r\theta$
	….. = 24 (cm^2)	A1	2	24; NMS scores 2/2
	Total		4	
	Condone absent or incorrect units in this question			

Q2	Solution	Mark	Total	Comment
(a)	$\frac{\sin C}{6} = \frac{\sin 120}{16}$	M1		OE eg next line in soln.
	$\sin C = \frac{6\sin 120}{16}$ $\left(= \frac{6\times 0.866}{16}\right)$	dM1		OE eg $\left(= \frac{6\sqrt{3}}{16\times 2}\right)$ or eg 0.3247…
	$C = 18.95... = 19^\circ$ (to nearest degree)	A1	3	Must see a value for C as 18.9 or AWRT 18.90 to 18.97 inclusive before seeing 19
(b)	(Method using angle B)			
	Angle B = 41 (.048...)	B1		41 or 41.0 or 41.04 or AWRT 41.05 Value may be seen on the diagram
	(Area=) $\frac{1}{2}\times 6\times 16\times \sin B$	M1		OE
	= 31.5 (cm^2) (to 3sf)	A1	3	CAO 31.5 only NMS scores 0/3
Alt 1	(Method not using angle B) $16^2 = 6^2 + AC^2 - 2(6)AC\cos 120$			
	$\Rightarrow AC = \sqrt{229} - 3$	(B1)		$\sqrt{229} - 3$ OE or 12.1 or 12.1… Value may be seen on the diagram
	(Area=) $\frac{1}{2}\times\left(\sqrt{229} - 3\right)\times 16\times \sin C$	(M1)		OE
	= 31.5 (cm^2) (to 3sf)	(A1)	(3)	CAO 31.5 only ; NMS scores 0/3
	Total		6	
 (a) (a)(b) (b)	Condone absent or incorrect units in this question Verification using 19: Dep on which formula is used, M1 or M1dM1 can be scored but then A0. If different labels are used for the angles, look for later evidence before applying any penalty; eg cand states a/sinA=b/sinB; 16/sin120=6/sinB; then correct rearrangement and calculation to B=18.95 = 19; In (b) cand states Area=1/2 absinC, finds C=41 and has 1/2x6x16xsin41=31.5. No penalty, (a)3/3 (b)3/3 unless contradiction eg check diagram does not have values 19 and 41 placed incorrectly at B and C. **NB** $0.5\times 6\times 16\times \sin(120+18.95) = 31.5$ scores 0/3			

Q3	Solution	Mark	Total	Comment
(a)	$\sqrt{27^x} = 3^{0.5(3x)}$	B1		Seen or used. eg $\log\sqrt{27^x} = 1.5x\log 3$.
	$\dfrac{\sqrt{27^x}}{3^{2x-1}} = 3^{0.5(3x)-(2x-1)}$	M1		$3^{kx} \div 3^{2x-1}$ OE$= 3^{kx-(2x-1)}$ or $= 3^{kx-2x-1}$ or $p\log 3 = kx\log 3 - (2x-1)\log 3$
	(= $3^{1-0.5x}$) (OE Accept form p = ……)	A1	3	OE 3^p Expression for p need not be simplified. eg $3^{0.5(3x)-(2x-1)}$ NMS 3/3
(b)	$\sqrt[3]{81} = 3^{\frac{4}{3}}$	B1		Seen or used; or $3^{3p} = 3^4$ or $\dfrac{\log 81}{\log 3} = 4$
	$3^{1-0.5x} = 3^{\frac{4}{3}} \Rightarrow x = -\dfrac{2}{3}$	B1	2	OE must be exact and from correct work. NMS scores 0/2
	Total		5	
(a)	Consult TL if cand has changed **both** numerator and denominator into form eg $9^{f(x)}$ then applied index/log law.			

Q4	Solution	Mark	Total	Comment
(a)	$u_1 = 108$; $u_2 = 72$	B1; B1	2	
(b)	$\{S_\infty =\}$ $\dfrac{a}{1-2/3}$ or $\dfrac{\text{c's } u_1}{1-r}$	M1		$\dfrac{\text{c's } u_1 \text{ value from (a)}}{1-r}$ or $\dfrac{a}{1-\frac{2}{3}}$
	$\dfrac{\text{c's } u_1 \text{ value from (a)}}{1-\frac{2}{3}}$	A1F		
	$\{S_\infty =\}$ 324	A1	3	Correct exact value for S_∞. NMS 324 scores 3 marks unless FIW.
(c)	$\sum_{n=k}^{\infty} u_n = S_\infty - \sum_{n=1}^{k-1} u_n$	M1		OE eg $\sum_{n=k}^{\infty} u_n = S_\infty - S_{k-1}$
	$324 - 324\left(1 - \left(\frac{2}{3}\right)^{k-1}\right) < 2.5$	A1F		Condone < replaced by either = or $\le$. OE If incorrect, ft on c's +'ve value for S_∞ from (b); ineq/eq can be in an unsimplified form but **only** unknown is k.
	(Smallest value of) k is 13	A1	3	**SC**1 mark for i) NMS k=13 ii) using $\sum_{n=k}^{\infty} u_n = S_\infty - S_k$ to get k=12
Alt 1	$\sum_{n=k}^{\infty} u_n = \dfrac{u_k}{1-r}$	(M1)		
	$486\left(\frac{2}{3}\right)^k < 2.5$	(A1)		OE Condone < replaced by either = or $\le$
	(Smallest value of) k is 13	(A1)	(3)	
	Total		8	
(a)(b)	Eg $u_1 = 108$; B1 $u_2 = 72$ B1; (b) $\dfrac{162}{1-r}$ (nothing yet) = $\dfrac{162}{1-2/3}$ M1 A0F = 486 A0			
(c)	$\sum_{n=k}^{\infty} u_n = S_\infty - S_n$ is M0, we need to see n replaced by $k-1$ before M1 can be awarded.			

(c)	k=13 with sufficient evidence eg $\frac{u_{13}}{1-r}=\frac{0.83239..}{1/3}=2.497...<2.5$ can score 3/3

Q5	Solution	Mark	Total	Comment
(a)	For st pt, $x^{\frac{3}{2}}-2x=0$	**M1**		$x^{\frac{3}{2}}-2x=0$
	$(\Rightarrow x^{\frac{3}{2}}=2x)$			
	(Since $x>0$) $\Rightarrow x^{\frac{1}{2}}=2\Rightarrow x=4$	**A1**	**2**	x=4 as the **only** value of x. [Give BOD if $x^3-4x^2=0$ appears after $x^{\frac{3}{2}}-2x=0$ in working]
(b)	$\frac{d^2y}{dx^2}=\frac{3}{2}x^{\frac{1}{2}}-2$	**M1** **A1**		Differentiating one term correctly. ACF
	When $x=4$, $\frac{d^2y}{dx^2}=1>0$ so curve has a minimum point	**A1**	**3**	AG Must be using 'hence'. Be convinced. eg shows that the value of the second derivative is 1 at x=4, states 1>0 (or states $\frac{d^2y}{dx^2}>0$) so min.
(c)	$\int(x^{1.5}-2x)dx=\frac{x^{2.5}}{2.5}-\frac{2x^2}{2}(+c)$	**M1**		Attempt to integrate $\frac{dy}{dx}$ with at least one of the two terms integrated correctly.
	$(y=)\ \frac{2}{5}x^{2.5}-x^2\ (+c)$	**A1**		$\frac{2}{5}x^{2.5}-x^2$ OE ; condone unsimplified
	When $x=4,\ y=2$ $\Rightarrow 2=\frac{2}{5}(4)^{2.5}-4^2+c$	**dM1**		Subst. $x=4$ or c's positive x value from part **(a)**, and y =2 into y = F(x)+'c' in an attempt to find the constant of integration
	$y=\frac{2}{5}x^{2.5}-x^2+\frac{26}{5}$	**A1**	**4**	ACF of the **equation** with signs and coefficients simplified
	Total		**9**	

Q6	Solution	Mark	Total	Comment
(a)(i)	$h = 0.25$	B1		$h = 0.25$ OE stated or used. (PI by x-values 0, 0.25, 0.5, 0.75, 1 provided no contradiction)
	$f(x) = 2^{3x}$ I $\approx \frac{h}{2}$ {f(0)+f(1)+2[f(0.25)+f(0.5)+f(0.75)]}	M1		$h/2${f(0)+f(1)+2[f(0.25)+f(0.5)+f(0.75)]} OE summing of areas of the 'trapezia'.. (**M0** if using an incorrect f(x).)
	$\frac{h}{2}$ with$\{\ldots\}=1+8+2\left(2^{\frac{3}{4}}+2^{\frac{3}{2}}+2^{\frac{9}{4}}\right)$ $=9+2(1.68...+2\sqrt{2}+4.756...)=$ $9+2\times 9.267... = 27.534\ldots$ (I $\approx \frac{0.25}{2}[27.534...]$) (= 3.4417…)	A1		OE Accept 2sf rounded or truncated or better evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term which rounds to 3.44
	= 3.44 (to 2 dp)	A1	4	CAO Must be 3.44 **SC** 5 strips used: **Max** B0M1A0; 3.41 A1
(a)(ii)	Increase the number of ordinates	E1	1	OE eg increase the number of strips
(a)(iii)	(Area=) $1\times k-\int_0^1 2^{3x}\,dx$	M1		PI by eg the next line
	$= 8 -$ c's answer to (a)(i)	dM1		Do **not** award if c's (a)(i) is ≥ 8
	= 4.56	A1	3	CAO Must be 4.56 **SC** 1 mark for final answer 4.56 coming from $3.44-8=-4.56$
(b)(i)	(Translation) $\begin{bmatrix}\frac{4}{3}\\0\end{bmatrix}$	B2,1,0	2	**B2** for $\begin{bmatrix}\frac{4}{3}\\0\end{bmatrix}$. If not **B2** award **B1** for $\begin{bmatrix}c\\0\end{bmatrix}$, where $c>0$ OE.
(b)(ii)	(Stretch) scale factor 2^{-4} in y-direction	B2,1,0	2	OE If not **B2** award **B1** for either correct sf or correct direction of stretch
(c)	$(3x-4)\log 2=\log 7$	M1		OE eg $3x-4=\log_2 7$
	$x = 2.2691\ldots.$ = 2.27 (to 3sf)	A1	2	2.27 Condone >3sf ie 2.269(118….) rounded or truncated If logs not used explicitly then 0/2.
	Total		**14**	

(a)(i)	For guidance, separate trapezia, 0.335(2…) + 0.563(7…) + 0.948(1..) + 1.594(6…)
(a)(i); (c)	If relevant brackets are missing, look at later work for further evidence of recovery.
(a)(ii)	Eg 'Use more decimal places' **E0**
(b)(i)	Must be given as a vector.

Q7	Solution	Mark	Total	Comment
(a)	(Area) = $\int_1^2 \left(7x+6-\frac{1}{x^2}\right)(\mathrm{d}x)$	B1		Area expressed as a correct definite integral. PI by fully correct integration and correct use of correct limits
	$\int \left(7x+6-\frac{1}{x^2}\right)(\mathrm{d}x)=\frac{7x^2}{2}+6x+x^{-1}$	M1 A1		Correct integration of two of the terms Correct integration of all 3 terms, can be left unsimplified.
	(Area) $=(\frac{28}{2}+12+2^{-1})-\left(\frac{7}{2}+6+1\right)$	M1		$\mathrm{F}(2)-\mathrm{F}(1)$, where $\mathrm{F}(x)$ is **not** the integrand
	$=26\frac{1}{2}-10\frac{1}{2}=16$	A1	5	AG Be convinced
(b)	Gradient of the line $2y+8x=3$ is -4	B1		-4 OE. PI by later work eg grad tang=1/4 Condone any errors in the rearrangement of constant term
	$\frac{\mathrm{d}y}{\mathrm{d}x}=7+2x^{-3}$	B1		
	At Q, $7+2x^{-3}=\frac{1}{4}$	M1		c's $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression = negative reciprocal of c's numerical gradient of given line OE
	$x^{-3}=-\frac{27}{8}$, $x=-\frac{2}{3}$	A1		Correct exact x -value
	$y=-\frac{11}{12}$	A1		Correct exact y -value
	Normal at Q: $y+\frac{11}{12}=-4\left(x+\frac{2}{3}\right)$	A1	6	ACF with signs simplified eg $12y+48x+43=0$
	Total		**11**	

Q8	Solution	Mark	Total	Comment
(a)	$\theta = 48^\circ$, 312°	B1 B1	 2	48 Condone 48.1… , 48.2 312 CAO Ignore values outside the given interval. If more than 2 values in given interval deduct 1 mark for each extra (to min of 0)
(b)(i)	$4\tan\theta\sin\theta = 4\dfrac{\sin\theta}{\cos\theta}\sin\theta$	M1		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **used**
	$= 4\dfrac{1-\cos^2\theta}{\cos\theta}$	dM1		Replacing $\sin^2\theta$ by $1-\cos^2\theta$ to either correctly express $4\tan\theta\sin\theta$ in terms of $\cos\theta$ or to obtain $4(1-\cos^2\theta) = \cos\theta(4-\cos\theta)$
	$(\cos\theta \neq 0)$ $4(1-\cos^2\theta) = \cos\theta(4-\cos\theta)$ $4 - 4\cos^2\theta = 4\cos\theta - \cos^2\theta$ $\Rightarrow 3\cos^2\theta + 4\cos\theta - 4 = 0$	A1	3	AG Be convinced.
(b)(ii)	$(\cos\theta + 2)(3\cos\theta - 2)$ (= 0)	B1		Correct factorisation or $\cos\theta = \dfrac{2}{3}, -2$
	Since $-1 \leq \cos\theta \leq 1$, $\cos\theta \neq -2$ so $\dfrac{2}{3}$ is the only value for $\cos\theta$.	E1	2	Valid explanation that would eliminate one of the c's values, with 'only one value' or an indication of which value is rejected.
(c)	$(\cos 4x \neq 0)$ $\cos 4x = \dfrac{2}{3}$	M1		$\cos 4x = \dfrac{2}{3}$. Ft on c's value in (b)(ii) provided $-1 \leq \cos\theta \leq 1$. PI eg by finding solns for $\cos\theta = \dfrac{2}{3}$ and clear attempt to divide values by 4
	$4x = 48^\circ, 312^\circ, 408^\circ, 672^\circ$	A1		$4x$ equal to or rounding to OE to the four integer values 48, 312, 408, 672 **seen**
	$(x =)\ 12^\circ,\ 78^\circ,\ 102^\circ, 168^\circ$	B2,1,0	4	If not **B2** award **B1** if either 2 correct or 3 AWRT three of these values. If more than four values in given interval, deduct 1 mark for each extra, to a min of **B0**. Ignore values outside $0^\circ \leq x \leq 180^\circ$. NMS Max 2/4.
	Total		**11**	
	Condone missing degree symbols NB Prem approx for 2/3 in (a) may lead to solns 49 and 311 (**B0 B0**); In (c), if the 49 is used for $4x$, then the same values for x should be obtained and we will award a possible max of **M1A0B2**.			
(b)(i)	Condone three non-zero terms written in a different order eg $4\cos\theta + 3\cos^2\theta - 4 = 0$.			
(b)(ii)	If using a letter for $\cos\theta$, the letter should be defined eg let $y = \cos\theta$, $(y+2)(3y-2)$ (=0) scores **B1**			
(b)(ii)	Explanation must be based on $-1 \leq \cos\theta \leq 1$ to justify the elimination of one invalid ft value from cand's			

	two values. Condone ‘between’ -1 and 1. Examples ‘ Math error’, ‘impossible’ ,‘can’t be negative’ **E0**

Q9	Solution	Mark	Total	Comment
	$\log_2(c+2)^3 - \log_2\left(\frac{c^3}{2}+k\right) = 1$	**M1**		$3\log(c+2) = \log(c+2)^3$
	$\log_2\left(\frac{(c+2)^3}{\left(\frac{c^3}{2}+k\right)}\right) = 1$	**M1**		Either $\log A - \log B = \log\frac{A}{B}$ or $1+\log_2 B = \log_2 2B$ used with correct *A* and *B*; if cand is using their expansion for $(c+2)^3$ in place of $(c+2)^3$, ignore any errors in the expansion in awarding this M1 mark
	$\log_2\left(\frac{(c+2)^3}{\left(\frac{c^3}{2}+k\right)}\right) = \log_2 2$	**B1**		$1 = \log_2 2$ stated or used **<u>at any stage</u>**. This also includes the step $\log_2 f(c,k) = p \Rightarrow f(c,k) = 2^p$.
	$(c+2)^3 = 2\left(\frac{c^3}{2}+k\right)$			(*) see below
	$c^3 + 6c^2 + 12c + 8$ $= 2\left(\frac{c^3}{2}+k\right)$	**B2,1,0**		$(c+2)^3 = c^3 + 6c^2 + 12c + 8$ seen **or used** at any stage; **B1** if 3 of the 4 terms are correct. May have to check correct collecting of like terms at a later stage in soln. [See below for altn for these two B marks]
	$\Rightarrow 6c^2 + 12c + 8 = 2k$	**A1**		OE Correct equation with no logs and no c^3 term.
	$\Rightarrow 6(c^2+2c+1) = 2k-2$ $\Rightarrow (c+1)^2 = \frac{2k-2}{6} = \frac{k-1}{3}$	**A1**	**7**	ACF for the expression in *k*.
	Total		**7**	

Altn: for the two B marks using the difference of two cubes ie $X^3 - Y^3 = (X-Y)(X^2+XY+Y^2)$

$(c+2)^3 - c^3 = (c+2-c)\{(c+2)^2 + (c+2)c + c^2\}$ **B1** (PI by next line)

$= 2(c^2+4c+4+c^2+2c+c^2)$ OE **B1**

(*) $\log(f(c,k)) = \log 2$, crossing out both ‘log’ to get $(f(c,k)) = 2$ we will condone

(*) $\log(f(c,k)) = \log 2$, $\frac{\log(f(c,k))}{\log 2} = 1$, $\frac{(f(c,k))}{2} = 1$ to get $(f(c,k)) = 2$ will result in FIW **A0 A0**