A-LEVEL

Mathematics

Pure Core 3 – MPC3

Mark scheme

6360

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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

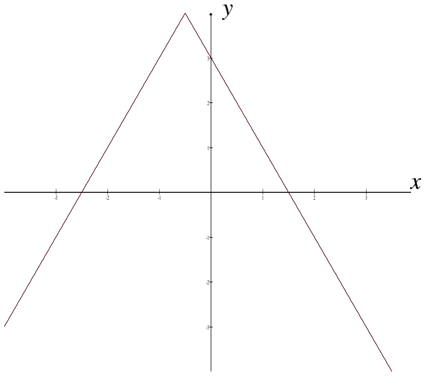
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
1a				
	<table><tr><td>x</td><td>y</td></tr><tr><td>2</td><td>$1\times\ln 4 = 1.38629$</td></tr><tr><td>3</td><td>$e^{-1}\times\ln 7 = 0.71586$</td></tr><tr><td>4</td><td>$e^{-2}\times\ln 10 = 0.31162$</td></tr><tr><td>5</td><td>$e^{-3}\times\ln 13 = 0.12770$</td></tr></table>	B1		All 4 correct x values (and no extras used) PI by 4 correct y values
		M1		At least 3 correct y in exact form or decimal values, rounded or truncated to 3dp or better (in table or formula) (PI by correct answer)
	$\int = (1\times)\sum y$	m1		Correct substitution into formula, with h=1 of 4, and only 4, correct y values (as above) either listed (with + signs) or totalled.
	$= 2.541$	A1	4	CAO, must be this exactly and no error seen
b	$\left(\frac{dy}{dx}=\right) -e^{2-x}\ln(3x-2)+e^{2-x}\frac{3}{3x-2}$	M1		$Ae^{2-x}\ln(3x-2)+e^{2-x}\frac{B}{3x-2}$
		A1		$A=-1$
		A1		$B=3$
	(When $x=2$) $\left(\frac{dy}{dx}=\right)\frac{3}{4}-\ln 4$ or $\frac{3}{4}+\ln\frac{1}{4}$	A1	4	ISW
	Total		**8**	

(a) NMS: An answer of 2.541 without anything else earns 0/4
The '1 x' may not be seen but implied

(b) NMS: An answer of -0.636 without anything else earns 0/4

Q2	Solution	Mark	Total	Comment
a		M1 A1		Correct shape, inverted V, roughly symmetrical, with vertex in the 2nd quadrant In all 4 quadrants
	(1.5, 0) **and** (-2.5, 0) (0, 3)	B1 B1	4	Shown on sketch or coordinates stated Shown on sketch or coordinates stated (diagram takes precedence)
b	$(x=)1$ $x=4+(2x+1)$ $(x=)-5$	B1 M1 A1	3	OE
c	$-5<x<1$	B2	2	Or for $x>-5$ **AND** $x<1$
d	Reflection in $y=k$ x-axis (or line $y=0$) (followed by) Translation $\begin{bmatrix}0\\p\end{bmatrix}$ $p=4$	M1 A1 M1 A1	4	Translation $\begin{bmatrix}0\\p\end{bmatrix}$ (M1) $p=4$ (A1) (followed by) (PI) Reflection in $y=k$ (M1) $k=4$ (A1) oe
	Total		13	

(a) For **M1** must be attempt at straight lines. Condone correct values on axes for **B1, B1**

(b) NMS: $x=-5$ scores **SC1**
If squaring: $x^2-8x+16=4x^2+4x+1$ therefore $3x^2+12x-15=0$ scores **M1**, then **A1, B1** as above

(c) $x>-5, x<1$ scores **SC1** $x>-5$ or $x<1$ scores **SC1**
SC1 for $-5\le x\le 1$ or $-5\le x\le 1$ or $-5\le x\le 1$

(d) There are other correct possible transformations, but for full marks the order of the two transformations must produce the correct answer.

Q3	Solution	Mark	Total	Comment
ai	$\mathrm{f}(x) = 6\ln x - 8x + x^2 + 3$ f(5) = - 2.3 f(6) = 1.75 Change of sign(or different signs) $\Rightarrow 5 < \alpha < 6$	M1 A1	 2	(or reverse) Both values correct to 1sf (rounded or truncated) Must have both statement and interval in words or symbols AND f(x) defined **OR** comparing 2 sides: $6\ln 5 = 9.7 \quad 8 \times 5 - 5^2 - 3 = 12$ $6\ln 6 = 11 \quad 8 \times 6 - 6^2 - 3 = 9$ (M1) at 5, LHS < RHS; at 6 LHS > RHS $\Rightarrow 5 < \alpha < 6$ (A1)
ii	$x = 4 + \sqrt{13 - 6\ln x}$ $x - 4 = \sqrt{13 - 6\ln x}$ $(x-4)^2 = 13 - 6\ln x$ $x^2 - 8x + 16 = 13 - 6\ln x$ $6\ln x + x^2 - 8x + 3 = 0$	M1 A1 A1	 3	Correctly eliminate square root Must see squared term correctly expanded AG, CSO
iii	$x_2 = 5.828$ $x_3 = 5.557$	B1 B1	2	
bi	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6}{x} + 2x - 8$ $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0\right) \; 6 + 2x^2 - 8x = 0$ $x = 1, \quad x = 3$ $(x = 1), \quad y = -4$ $(x = 3), \quad y = 6\ln 3 - 12$ **or** $\ln 729 - 12$	B1 M1 A1 A1 A1	 5	Condone $\dfrac{6x^5}{x^6}$ Equate to zero (PI) and eliminate their fraction correctly. Oe for other exact correct values If **M0** then SC1 for (1, -4) and/or $(3, 6\ln 3 - 12)$
ii	$x = 5, \quad y = -8$ $x = 7, \quad y = 12\ln 3 - 24$	M1 A1	 2	their x + 4 and $2 \times$ their y on either of their 'pairs' All correct : oe exact
	Total		**14**	

(a)(ii) Condone all terms in any order on one side but must have =0
(a)(iii) No credit for any answers not to this accuracy

Q4	Solution	Mark	Total	Comment
a		M1		$f(x) \le 5$, $** < 5$
	$f(x) < 5$	A1	2	
bi	$x = 5 - e^{3y}$	M1		Swap x and y at any stage.
	$e^{3y} = 5 - x$			
	$3y = \ln(5 - x)$	M1		Correctly converting to ln.
	$(f^{-1}(x) =)\dfrac{1}{3}\ln(5 - x)$	A1	3	ACF
ii	$(x =)\ \ 4$	B1	1	
c	$[gg(x) =]\ \dfrac{1}{2(\frac{1}{2x-3}) - 3}$	M1		
	$= \dfrac{1}{\frac{2 - 6x + 9}{2x - 3}}$	A1		or $\dfrac{2x-3}{2-3(2x-3)}$
	$= \dfrac{2x-3}{11-6x}$	A1	3	
	Total		9	

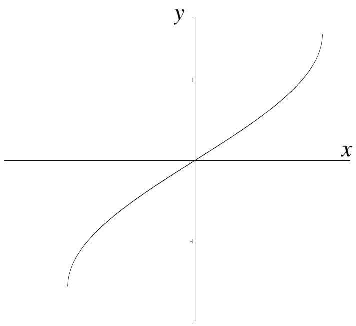
(b)(i) Must be convinced that final answer is not $\ln\dfrac{5-x}{3}$ or $\ln(5-x)/3$

Q5	Solution	Mark	Total	Comment
a	$(\frac{dy}{dx}=)\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$	M1		$\frac{\pm\cos^2 x\pm\sin^2 x}{\cos^2 x}$
	$=\frac{1}{\cos^2 x}$ or $1+\tan^2 x$			Must see this line
	$=\sec^2 x$	A1	2	AG; no errors seen and all notation correct
b	$\int x\sec^2 x\mathrm{d}x$			
	$u=x$ $\quad \frac{\mathrm{d}v}{(\mathrm{d}x)}=\sec^2 x$			
	$\frac{\mathrm{d}u}{(\mathrm{d}x)}=1$ $\quad v=\tan x$	M1		All 4 terms in this form with $\frac{du}{dx}$ correct and $\int\frac{\mathrm{d}v}{\mathrm{d}x}$ attempted
	$v=\tan x$	B1		
	$x\tan x-\int\tan x(\mathrm{d}x)$	A1		
	$=x\tan x-\ln\sec x+c$	A1	4	OE (e.g. $x\tan x+\ln\cos x$); must have constant of integration
c	$(\mathrm{V}=)\pi\int_0^1 25x\sec^2 x\mathrm{d}x$	B1		Must include π, limits and dx
	$=(25\pi)[(1\tan 1-\ln\sec 1)-0]$	M1		Must have $(k)\int x\sec^2 x$ then correct substitution of 0 and 1 into $ax\tan x+b\ln(\sec\ or\ \cos)x$ Condone missing 0.
	$=74$	A1	3	Condone AWRT 74
	Total		9	

(a) Use of product rule scores **M0**

(c) $\left[(5\sqrt{x})\sec x\right]^2$ must be correctly expanded for B1 to be available.

If the integration has been re-started, then **M1** must be for substitution into $ax\tan x+b\ln\sec x$

Q6	Solution	Mark	Total	Comment
a		B1		Correct shape passing through origin
	$\left(\frac{1}{3}, \frac{\pi}{2}\right)$	B1		Must be stated
	$\left(-\frac{1}{3}, -\frac{\pi}{2}\right)$	B1	3	Must be stated
b	$\frac{dx}{dy} = \frac{1}{3}\cos y$	M1		
	$\frac{dy}{dx} = \frac{3}{\cos y}$ or $3\sec y$	A1	2	Both $\frac{dx}{dy}$ and $\frac{dy}{dx}$ seen and used correctly
	Total		**5**	

(a) Coordinates must be stated NOT just indicated on axes, but BOTH correct end points clearly labelled on axes scores **SC1.**

Q7	Solution	Mark	Total	Comment
	$\frac{\mathrm{d}u}{\mathrm{d}x} = -2x$ or $\mathrm{d}u = -2x\,\mathrm{d}x$	M1		Condone $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ or $\mathrm{d}u = 2x\,\mathrm{d}x$
	$\int \frac{6-u}{u^{0.5}} \times \frac{\mathrm{d}u}{-2}$	A1		OE correct unsimplified integral in terms of u only, with du seen on this line or later
	$= -\frac{1}{2}\int (6u^{-0.5} - u^{0.5})\mathrm{d}u$	m1		Terms in the form $\int (a\,u^{-0.5} + bu^{0.5})\mathrm{d}u$
	$= -\frac{1}{2}(6\frac{u^{0.5}}{0.5} - \frac{2u^{1.5}}{3})$	A1F		Ft must be in the form $cu^{0.5} + du^{1.5}$ Oe (eg allow $c\sqrt{u}$)
	$(= -6u^{0.5} + \frac{1}{3}u^{1.5})$			
	(Limits $[x]_1^2 = [u]_5^2$)			
	$\int_5^2 = \left[-6u^{0.5} + \frac{1}{3}u^{1.5} \right]_5^2$ $= (-6\times 2^{0.5} + \frac{1}{3}\times 2^{1.5}) - (-6\times 5^{0.5} + \frac{1}{3}\times 5^{1.5})$	m1		Correct substitution into expression of the form $eu^{0.5} + fu^{1.5}$ and F(2) – F(5), or if using x, F(2) – F(1)
	$= \frac{13}{3}\sqrt{5} - \frac{16}{3}\sqrt{2}$	A1A1	7	oe any correct exact form
	Total		**7**	

For first A1 allow: $\int \frac{(6-u)^{\frac{3}{2}}}{\sqrt{u}(6-u)^{\frac{1}{2}}} \times \frac{du}{-2}$

For second m1 the substitution must be in the correct order

Q8	Solution	Mark	Total	Comment
a	$LHS = 4(1+\cot^2\theta) - \cot^2\theta$	M1		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$4(1+\cot^2\theta) - \cot^2\theta = k$ Or $4\operatorname{cosec}^2\theta - (\operatorname{cosec}^2 - 1) = k$	A1		All correct, including = k
	$\cot^2\theta = \dfrac{k-4}{3}$	m1		Correctly isolating trig function – must be tan or cot or cos or sec, from their **CORRECT** equation
	$\tan^2\theta = \dfrac{3}{k-4}$	m1		Correct inversion (at some stage) from their equation
	$[\sec^2\theta = \dfrac{3}{k-4} + 1]$			Must see at least one line of working, be convinced
	$\sec^2\theta = \dfrac{k-1}{k-4}$	A1	5	AG: no errors seen
b	$\sec^2\theta = 4$ or $\tan^2\theta = 3$ or $\cot^2\theta = \dfrac{1}{3}$ or $\operatorname{cosec}^2\theta = \dfrac{4}{3}$	B1		PI by expression for eg $\sec x = 2$
	$\sec\theta = \pm 2$	M1		or $\cos\theta = \pm 0.5$ or $\tan\theta = \pm\sqrt{3}$ or $\sin\theta = \pm\dfrac{\sqrt{3}}{2}$
	$(\theta =)$ 60, 120, 240, 300, 420	A1		Sight of any four of these answers
	$x = 22.5°, 82.5°, 112.5°, 172.5°$	B1 B1	5	3 correct All correct and no extras in interval (ignore answers outside interval)
	Total		**10**	

(a) The two **m1** marks can be earned in either order.
There are many different approaches

(b) If working in radians then max mark is **B1, M1**

(a)	Different approaches: **I** $LHS = 4(1+\cot^2\theta)-\cot^2\theta$	**M1**		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$4(1+\cot^2\theta)-\cot^2\theta = k$	**A1**		All correct, including $= k$
	$k-1 = 3+3\cot^2\theta$ $k-4 = 3\cot^2\theta$	**m1**		Both correct equations from their equation in k.
	$\dfrac{k-1}{k-4} = \dfrac{3+3\cot^2\theta}{3\cot^2\theta}$	**m1**		Correct equation from their 2 previous equations
	$\sec^2\theta = \dfrac{k-1}{k-4}$	**A1**		AG: no errors seen
	II $LHS = \dfrac{4}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$ $= \dfrac{4-\cos^2\theta}{1-\cos^2\theta}$	**M1**		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$\dfrac{4-\cos^2\theta}{1-\cos^2\theta} = k$	**A1**		All correct, including = k
	$\dfrac{4\sec^2\theta-1}{\sec^2\theta-1} = k$	**m1**		Correct 'inversion' (at some stage) from their equation
	$4\sec^2\theta - 1 = k\sec^2\theta - k$			Must see at least one line of working, be convinced for final A1
	$k-1 = \sec^2\theta(k-4)$	**m1**		Correct equation in the form $a\sec^2\theta = b \quad or \quad a\cos^2\theta = b$ from their **CORRECT** equation
	$\sec^2\theta = \dfrac{k-1}{k-4}$	**A1**		AG: no errors seen