AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Pure Core 3

Wednesday 15 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $y = (4x + 1)^3 \sin 2x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) Given that $y = \dfrac{2x^2 + 3}{3x^2 + 4}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{px}{(3x^2 + 4)^2}$, where p is a constant.

[2 marks]

(c) Given that $y = \ln\left(\dfrac{2x^2 + 3}{3x^2 + 4}\right)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	**Answer space for question 1**

2 The curve with equation $y = x^x$, where $x > 0$, intersects the line $y = 5$ at a single point, where $x = \alpha$.

(a) Show that α lies between 2 and 3.

[2 marks]

(b) Show that the equation $x^x = 5$ can be rearranged into the form

$$x = e^{\left(\frac{\ln 5}{x}\right)}$$

[3 marks]

(c) Use the iterative formula

$$x_{n+1} = e^{\left(\frac{\ln 5}{x_n}\right)}$$

with $x_1 = 2$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(d) (i) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to

$$\int_{0.5}^{1.7} (5 - x^x)\,dx$$

giving your answer to three significant figures.

[4 marks]

(ii) Hence find an approximation to $\int_{0.5}^{1.7} x^x\,dx$.

[2 marks]

QUESTION PART REFERENCE

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3 Solve

$$x^2 \geqslant |5x - 6|$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	**Answer space for question 3**

4 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x-5}$.

[4 marks]

(b) The **normal** to the curve $y = e^{2x-5}$ at the point $P(2, e^{-1})$ intersects the x-axis at the point A and the y-axis at the point B.

Show that the area of the triangle OAB is $\frac{(e^2+1)^m}{e^n}$, where m and n are integers.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 4

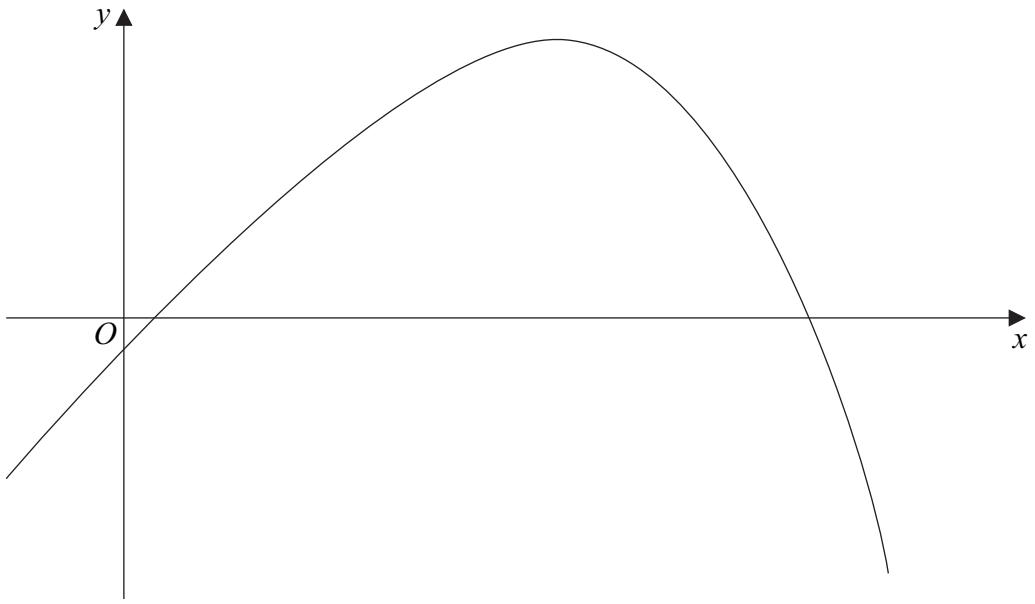
QUESTION PART REFERENCE

Answer space for question 4

5 The function f is defined by

$$\mathrm{f}(x) = 16x - \mathrm{e}^{2x}, \quad \text{for all real } x$$

The graph of $y = \mathrm{f}(x)$ is sketched below.



(a) Find the range of f.

[5 marks]

(b) The composite function fg is defined by

$$\mathrm{fg}(x) = \frac{16}{x} - \mathrm{e}^{\frac{2}{x}}, \quad \text{for real } x,\ x \neq 0$$

Find an expression for $\mathrm{gg}(x)$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

6 (a) Use integration by parts to find $\int \frac{\ln(3x)}{x^2}\,\mathrm{d}x$.

[4 marks]

(b) The region bounded by the curve $y = \frac{\ln(3x)}{x}$, the x-axis from $\frac{1}{3}$ to 1, and the line $x = 1$ is rotated through 2π radians about the x-axis to form a solid.

Find the exact value of the volume of the solid generated.

[7 marks]

QUESTION PART REFERENCE

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7 (a) By writing $\sec x = (\cos x)^{-1}$, use the chain rule to show that, if $y = \sec x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec x \tan x$$

[2 marks]

(b) The function f is defined by

$$\mathrm{f}(x) = 2\tan x - 3\sec x, \quad \text{for} \quad 0 < x < \frac{\pi}{2}$$

Find the value of the y-coordinate of the stationary point of the graph of $y = \mathrm{f}(x)$, giving your answer in the form $p\sqrt{q}$, where p and q are integers.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE

Answer space for question 7

8 Use the substitution $u = 4x - 1$ to find the exact value of

$$\int_{\frac{1}{4}}^{\frac{1}{2}} (5 - 2x)(4x - 1)^{\frac{1}{3}} \, \mathrm{d}x$$

[7 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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9 (a) It is given that $\sec x - \tan x = -5$.

(i) Show that $\sec x + \tan x = -0.2$.

[2 marks]

(ii) Hence find the exact value of $\cos x$.

[3 marks]

(b) Hence solve the equation

$$\sec(2x - 70°) - \tan(2x - 70°) = -5$$

giving all values of x, to one decimal place, in the interval $-90° \leqslant x \leqslant 90°$.

[3 marks]

QUESTION PART REFERENCE

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END OF QUESTIONS

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