



A-LEVEL

Mathematics

MPC4 – Pure Core 4

Mark scheme

6360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$19x - 3 = A(3 - 4x) + B(1 + 2x)$ Correct equation and 'attempt' to find A or B $A = -\frac{5}{2}$ $B = \frac{9}{2}$	M1 A1 A1	3	e.g. Using $x = \frac{3}{4}$ or $-\frac{1}{2}$ or simultaneous equation such as $19 = -4A + 2B$ and $-3 = 3A + B$
	NMS or cover up rule scores SC2 for $A = -\frac{5}{2}$ or $B = \frac{9}{2}$ or SC3 for both $A = -\frac{5}{2}$ and $B = \frac{9}{2}$			
(b)(i)	$(1 + 2x)^{-1} = 1 - 2x + kx^2$ $= 1 - 2x + 4x^2$ $(3 - 4x)^{-1} = 3^{-1} \left(1 - \frac{4}{3}x\right)^{-1}$ $\left(1 - \frac{4}{3}x\right)^{-1}$ $= 1 + (-1) \left(-\frac{4}{3}x\right) + \frac{(-1)(-2)}{2!} \left(-\frac{4}{3}x\right)^2$ $= \left(1 + \frac{4}{3}x + \frac{16}{9}x^2\right)$ $\frac{19x - 3}{(1 + 2x)(3 - 4x)} =$ $-\frac{5}{2}(1 - 2x + 4x^2) + \frac{9}{2} \cdot \frac{1}{3} \left(1 + \frac{4}{3}x + \frac{16}{9}x^2\right)$ $= -1 + 7x - \frac{22}{3}x^2$	M1 A1 B1 M1 A1 M1 A1	7	provided $k \neq 0$ Accept $+(2x)^2$ ACF for 3^{-1} eg $\frac{1}{3}$ or 0.33 ...or 0.3 Condone poor use of or missing brackets. PI by later work PI by correct answer ft on candidate's A and B with their relevant series. Must have $\frac{22}{3}, 7\frac{1}{3}$ or $7.\dot{3}$
	Alt. $(3 - 4x)^{-1} = 3^{-1} + (-1)3^{-2}(-4x) + \frac{(-1)(-2)}{2!}3^{-3}(-4x)^2$ M1 $= \frac{1}{3} + \frac{4}{9}x + \frac{16}{27}x^2$ A2 If A and B are correct full expansions are $-\frac{5}{2} + 5x - 10x^2$ and $\frac{3}{2} + 2x + \frac{8}{3}x^2$ Alt. for combined expansions without using PF's. $\frac{19x-3}{(1+2x)(3-4x)} = (19x-3)(1+2x)^{-1}(3-4x)^{-1} = (19x-3)\left(1-2x+4x^2\right)\left(\frac{1}{3}+\frac{4}{9}x+\frac{16}{27}x^2\right)$ Attempt to multiply any two of their three brackets together as far as the term in x^2 M1 then A1 Condone unsimplified fractions in the binomial expansion(s), but final answer must be fully simplified. A and B could be included in the series - e.g. $\frac{9}{2(3-4x)} = \frac{9}{6-8x} = \frac{9}{6}\left(1 - \frac{8x}{6}\right)^{-1}$ could still score B1 M1 A1 .			
(b)(ii)	$-\frac{1}{2} < x < \frac{1}{2}$ only	B1	1	Strictly $<$: Accept $ x < \frac{1}{2}$
	Do NOT accept $-1 < 2x < 1$ or $ 2x < 1$ or inclusion of the equality sign. If $-\frac{3}{4} < x < \frac{3}{4}$ is also seen, it must be clear that $-\frac{1}{2} < x < \frac{1}{2}$ is the answer.			
	Total		11	

Q2	Solution	Mark	Total	Comment
	$\cos 2\theta = 2\cos^2\theta - 1$ used $3(2\cos^2\theta - 1) - 5\cos\theta + 2 = 0$ $6\cos^2\theta - 5\cos\theta - 1 = 0$ $(\cos\theta - 1)(6\cos\theta + 1) = 0$ $(\cos\theta = 1) \quad \cos\theta = -\frac{1}{6}$ $\theta = 99.6^\circ, 260.4^\circ$	B1 M1 m1 A1 A1		PI: Correct expression in terms of $\cos\theta$ used. Attempt to use identity for $\cos 2\theta$ of the form $a\cos^2\theta + b$ to obtain a quadratic in $\cos\theta$. Attempt to factorise their quadratic or correct use of quadratic formula. Either correct – CAO Both correct and no extra values in the interval but ignore any values outside of the interval including 0° and 360° .
	Total		5	
	To earn the m1 mark, candidate's factors must give their $6\cos^2\theta$ and their -1 i.e. the first and last terms of their quadratic. If the quadratic formula is used it must be used correctly for their quadratic. If they get the correct quadratic and NMS , both correct answers for $\cos\theta$ implies the m1 mark, or If they get the correct quadratic and NMS , one correct answer for θ implies the m1 mark. If they get a wrong quadratic, they must show the working to (possibly) score the m1 mark. Interval is specified to be $0^\circ < \theta < 360^\circ$; hence the reason for 'ignoring' solutions 0° and 360° that might come from $\cos\theta = 1$. Degree signs are not required; 99.6 and 260.4 are sufficient for the A marks. Allow SC1 if both rounded correctly to greater accuracy 99.5940... and 260.4059... if A0 A0 .			

Q3	Solution	Mark	Total	Comment
(a)	$\frac{3 + 13x - 6x^2}{2x - 3} = Ax + B + \frac{C}{2x - 3}$ $3 + 13x - 6x^2 = Ax(2x - 3) + B(2x - 3) + C$ Correct above equation and attempt to find one of A, B or C or an attempt at long division $A = -3$ $B = 2$ $C = 9$	M1 A1 A1 A1	4	e.g. using $x = \frac{3}{2}$ in an attempt to find C or forming simultaneous equations and attempt to solve.
	If long division is used award M1 once $-3x + \dots$ has been obtained but only award the A marks once the values are clearly identified or it is written in the required form of $Ax + B + \frac{C}{2x-3}$. Alternative method of division $\frac{3+13x-6x^2}{2x-3} = \frac{-3x(2x-3)-9x+13x+3}{2x-3} = -3x + \frac{4x+3}{2x-3}$ M1 for $-3x + \dots$. For the A marks, A, B and C must be clearly identified or seen in the required form of $Ax + B + \frac{C}{2x-3}$. NMS scores B2 for one correct value, B3 for 2 correct values and B4 for all three correct values			
(b)	$\int \frac{3+13x-6x^2}{2x-3} dx = \int -3x + 2 + \frac{9}{2x-3} dx$ $= px^2 + qx + r \ln(2x - 3)$ $= -\frac{3}{2}x^2 + 2x + \frac{9}{2}\ln(2x - 3)$ Correct use of $F(6) - F(3)$ $= \left[-\frac{3}{2} \cdot 6^2 + 2 \cdot 6 + \frac{9}{2} \ln(12 - 3) \right]$ $- \left[-\frac{3}{2} \cdot 3^2 + 2 \cdot 3 + \frac{9}{2} \ln(6 - 3) \right]$ $= -\frac{69}{2} + \frac{9}{2} \ln 3$	M1 A1ft m1 A1	4	$\frac{A}{2}x^2 + Bx + \frac{C}{2}\ln(2x - 3)$ Correct substitution of limits for their p, q and r. OE
	The M1 A1ft and m1 can be earned even if left in terms of A, B and C or if 'invented' value(s) for A, B and C are used. Condone missing brackets from the $\ln(2x - 3)$ term for the M1 mark but only award the A1ft mark if they have clearly recovered; PI by sight of $\ln 9$ or $\ln 3$ after using the limits or a correct final answer. Treat a decimal answer (should be $-29.55 \dots$) after a correct exact form as ISW but award A0 if an exact answer is not seen.			
	Total		8	

Q4	Solution	Mark	Total	Comment
(a)(i)	$m = m_0 k^t$ Using $m = 12, m_0 = 24$ and $t = 8$ $k^8 = \frac{1}{2} \quad \text{or} \quad k = (\sqrt[8]{0.5})$ $= 0.917004$	B1	1	$12 = 24k^8$ OE e.g. $k = \left(\frac{1}{2}\right)^{\frac{1}{8}}$ Must see a correct exact expression for k or k^8 or $k=0.91700404(32\dots)$ to at least 8 d. p. AG be convinced
	Note that AG so to earn the mark they must show us a correct exact expression for k or k^8. Accept such as $k = e^{\left(\frac{\ln 0.5}{8}\right)}$ or $e^{-0.086643\dots}$ or $\left(\frac{1}{2}\right)^{\frac{1}{8}}$ or $0.91700404(32\dots)$ as sufficient evidence but withhold the mark if a clear error has been made – e.g. $k = \sqrt[8]{0.5}$. Candidates who work with logs must reach an expression such as $\log k = \frac{\log 12 - \log 24}{8}$ first.			
(a)(ii)	$1 = m_0(0.917004)^{60}$ $m_0 = 181$	M1 A1	2	or $m_0 = (0.917004)^{-60}$ PI by A1 later Must be 181 no ISW
	NMS scores SC2 for 181 only but sight of greater accuracy (181.0198...) implies M1 if 181 not seen.			
(b)	$m = m_0 k^t$ $8.106 = 10 \times k^{100}$ $k = \sqrt[100]{0.8106} \quad \text{OE}$ $\frac{1}{2}m_0 = m_0 k^t$ $k^t = \frac{1}{2}$ $t \log k = \log\left(\frac{1}{2}\right)$ $t = \frac{\log\left(\frac{1}{2}\right)}{\log k}$ $= 330$	M1 A1 M1 A1	4	OE: e.g. $k = e^{\ln(0.8106)/100}$ A linear equation in t from $k^t = \frac{1}{2}$ e.g. $t = \log_k(0.5)$ Must be 330 No ISW
	For guidance, for first A1, $k = 0.9979\dots$ PI by later correct work. The first M1 is for a correct interpretation of the information given so could equally be awarded for an expression involving logs of k such as $\ln 8.106 = \ln 10 + 100 \ln k$ then A1 for a correct expression for $\ln k$ such as $\ln k = \frac{\ln 8.106 - \ln 10}{100}$ or, using base 10, $\log k = \frac{\log 8.106 - 1}{100}$. Those who use the value of k from (a) could only score M0 A0 M1 A0. NMS scores SC4 for 330 only but sight of greater accuracy (330.1006...) implies M1 A1 M1 if 330 not seen.			
	Total		7	

Q5	Solution	Mark	Total	Comment
(a)(i)	Use of $\sin^2 B + \cos^2 B = 1$ $\left(\frac{1}{\sqrt{5}}\right)^2 + \cos^2 B = 1$ $\cos B = \frac{2}{\sqrt{5}}$	B1	1	Or use of right-angled triangle with opp = 1 and hyp = $\sqrt{5}$ to get adj = $\sqrt{4}$ or 2. $\cos(\sin^{-1}(\frac{1}{\sqrt{5}})) = \frac{2}{\sqrt{5}}$ is B0 AG ; must see evidence of working
(a)(ii)	$(\sin 2B = 2 \sin B \cos B)$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	M1 A1	 2	Correct identity (PI) and substitution AG so line above must be seen.
(b)(i)	$\cos A = \frac{2}{3}$ exact value $\sin(A - B) = \sin A \cos B - \cos A \sin B$ $= \frac{\sqrt{5}}{3} \times \frac{2}{\sqrt{5}} - \frac{2}{3} \times \frac{1}{\sqrt{5}}$ Use of $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ or $\frac{10-2\sqrt{5}}{15}$ OE seen $\frac{2}{15}(5 - \sqrt{5})$	B1 M1 m1 A1	 4	$\cos A = \frac{2}{3}$ seen or used (not 0.667 etc.) ft on their value of $\cos A$ $\frac{2}{3\sqrt{5}}$ term becoming $\frac{2\sqrt{5}}{15}$ before final answer $\frac{2}{15}$ OE seen and be convinced
You must see justification between the use of the identity and the final answer to earn the m1 A1 .				
(b)(ii)	$\cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \frac{2}{3} \times \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3} \times \frac{1}{\sqrt{5}}$ $= \frac{1}{3} + \frac{4}{15} \sqrt{5}$	M1 A1 A1	 3	ft on their value of $\cos A$ fully correct OE for $\frac{1}{3}$ and $\frac{4}{15}$ but not left as $\frac{5+4\sqrt{5}}{15}$
Total			10	

Q6	Solution	Mark	Total	Comment
	NO MISREADS ARE ALLOWED IN THIS QUESTION			
(a)	$\overrightarrow{AB} = \begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$ $\overrightarrow{AB} \cdot \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} = (4 \times 3) + (-12 \times -5) + (-20 \times 1)$ $52 = \sqrt{4^2 + 12^2 + 20^2} \sqrt{3^2 + 5^2 + 1^2} \cos \theta$ $\cos \theta = \frac{52}{\sqrt{560} \sqrt{35}}$ $= \frac{13}{35}$	B1 B1ft M1 A1 A1	 5	or $\overrightarrow{BA} = \begin{bmatrix} -4 \\ 12 \\ 20 \end{bmatrix}$ Correctly finding scalar product using their $\overrightarrow{AB}$ and direction vector of l_2 Accept $12 + 60 - 20$ or 52 Correct use of their $\mathbf{a} \cdot \mathbf{b} = a b \cos \theta$ OE all correct in this form or better. CAO
	The B1 mark for $\overrightarrow{AB}$ or $\overrightarrow{BA}$ or any multiple could be PI by its use in the scalar product e.g. $\begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$ etc. The B1ft mark is for the scalar product of their $\overrightarrow{AB}$ with the direction vector of l_2. The M1 mark is for a clear attempt at the scalar product definition of "$\mathbf{a} \cdot \mathbf{b} = a b \cos \theta$" in any form using their $\overrightarrow{AB}$ with the direction vector of l_2. As in the MS, there is no need for minus signs in squared terms. Provided they earn the M1 mark with the correct values included, it is possible to score both A1 marks for $\cos \theta = \frac{13}{35}$ without the intermediate form being seen.			
(b)	Line AB: $(\mathbf{r} =) \begin{bmatrix} 0 \\ 6 \\ 9 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$ $\begin{aligned} -1 + 3\lambda &= 0 + 4\mu \\ 5 - 5\lambda &= 6 - 12\mu \\ -2 + \lambda &= 9 - 20\mu \end{aligned}$ $\lambda = 1 \quad \mu = \frac{1}{2}$ Verifying that all 3 three equations are satisfied and conclusion – e.g. ‘intersect’. $P(2, 0, -1)$	M1 A1A1 E1 A1	 5	Set up two correct equations for their l_1 but with correct l_2 and attempt to eliminate λ or μ . Accept these three equations in column vector form Clear checking of λ and μ in unused equation or showing P lies on both lines. Dependent on correct co-ordinates of P . Accept as a column vector
	Look out for any alternative correct versions for the vector equation of l_1, e.g. If $B(4, -6, -11)$ is used as known point in l_1 this leads to $\lambda = 1$ and $\mu = -\frac{1}{2}$ but also look out for ‘multiples’ of $\begin{bmatrix} 4 \\ -12 \\ -20 \end{bmatrix}$ being used as the direction vector.			

Q6	Solution	Mark	Total	Comment
(c)	Using points C and D $\overrightarrow{CD} = t \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$ $ \overrightarrow{AB} = 3 \overrightarrow{CD} \Rightarrow \sqrt{560} = 3\sqrt{35}t^2$ $t = (\pm) \frac{4}{3}$ $(\overrightarrow{OC/D}) = \overrightarrow{OP} + \frac{t}{2} \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + \frac{2}{3} \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$ $\left(4, -\frac{10}{3}, -\frac{1}{3}\right)$ $\left(0, \frac{10}{3}, -\frac{5}{3}\right)$	B1 B1 M1 A1 A1	5	Since C and D lie on l_2 or $\sqrt{560} = 3t\sqrt{35}$ $\frac{4}{3}$ seen Uses P the mid-point of AB and CD in this method. Use of their $\overrightarrow{OP}$ and $\frac{t}{2} \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$ Accept as a column vector. Accept as a column vector.
	Candidate could just get $t = \frac{4}{3}$ giving point $\left(4, -\frac{10}{3}, -\frac{1}{3}\right)$ and then find $\left(0, \frac{10}{3}, -\frac{5}{3}\right)$ by symmetry.			
(c)	R is any point on l_2 $(\overrightarrow{OR}) = \begin{bmatrix} -1 + 3\lambda \\ 5 - 5\lambda \\ -2 + \lambda \end{bmatrix}$ $(\overrightarrow{PR}) = \begin{bmatrix} -1 + 3\lambda \\ 5 - 5\lambda \\ -2 + \lambda \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ If R is at C or D then $\overrightarrow{AB} = 3 \overrightarrow{CD} = 6 \overrightarrow{PR}$ $\sqrt{560} = 6\sqrt{(-3 + 3\lambda)^2 + (5 - 5\lambda)^2 + (-1 + \lambda)^2}$ $560 = 36(35\lambda^2 - 70\lambda + 35)$ $9\lambda^2 - 18\lambda + 5 = 0$ $(3\lambda - 1)(3\lambda - 5) = 0$ $\lambda = \frac{1}{3} \quad \text{or} \quad \frac{5}{3}$ $\left(0, \frac{10}{3}, -\frac{5}{3}\right)$ $\left(4, -\frac{10}{3}, -\frac{1}{3}\right)$	(B1) (B1) (M1) (A1) (A1)	(5)	$\begin{bmatrix} -3 + 3\lambda \\ 5 - 5\lambda \\ -1 + \lambda \end{bmatrix}$ but award at $\overrightarrow{OR} - \overrightarrow{OP}$ stage or better Factorising (their $9\lambda^2$ and $+5$ terms) or attempting to solve their quadratic equation correctly – PI by correct values of λ. Accept as a column vector. Accept as a column vector.
	Total		15	

Q7	Solution	Mark	Total	Comment
(a)	$\left(\frac{dx}{dt}\right) = -(-6)\frac{e^{2-6t}}{4}$ $\left(\frac{dy}{dt}\right) = \frac{pe^{3t}.t + qe^{3t}}{(3t)^2}$ $= \frac{3e^{3t}.3t - e^{3t}.3}{(3t)^2}$ $\frac{dy}{dx} = \frac{(9.\frac{2}{3}.e^2 - 3e^2)/4}{\frac{3}{2}e^{-2}}$ $= \frac{1}{2}e^4$	B1 M1 A1 m1 A1	5	ACF : $\frac{3}{2}e^{2-6t}$ From quotient rule ACF : $\frac{9te^{3t}-3e^{3t}}{9t^2}$, $\frac{e^{3t}}{t} - \frac{e^{3t}}{3t^2}$ etc. Using $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ and clear evidence of an attempt to substitute $t = \frac{2}{3}$ (must be this) done in either order. CAO : Accept this or $\frac{e^4}{2}$ or $0.5e^4$ only
	$\frac{dy}{dt}$ found using product rule : $\frac{dy}{dt} = pe^{3t}.t^{-1} + qe^{3t}.t^{-2}$ M1 $= e^{3t}t^{-1} - \frac{1}{3}e^{3t}t^{-2}$ A1 $(= \frac{e^{3t}}{t} - \frac{e^{3t}}{3t^2})$ For guidance, when $t = \frac{2}{3}$, $\frac{dx}{dt} = \frac{3}{2}e^{-2}$ and $\frac{dy}{dt} = \frac{3}{4}e^2$			
(b)	$x = \frac{4-e^{2-6t}}{4} \Rightarrow 4x = 4 - e^{2-6t}$ leading to $e^{-6t} = \frac{4-4x}{e^2}$ $e^{6t} = \frac{e^2}{4(1-x)}$ $e^{3t} = \frac{e}{2\sqrt{1-x}}$	M1 A1	2	Any correct expression for e^{-6t} or e^{6t} . AG Must see inversion step and be convinced they haven't worked backwards
	Alternative for the M1 mark is a correct expression for $3t$ or e^{1-3t} or $\frac{1}{2}e^{1-3t}$ OE. $4x = 4 - e^{2-6t} \Rightarrow 2 - 6t = \ln(4 - 4x) \Rightarrow 3t = 1 - \frac{1}{2}\ln(4 - 4x)$ M1 or $4x = 4 - e^{2-6t} \Rightarrow e^{2-6t} = 4 - 4x \Rightarrow e^{1-3t} = \sqrt{4 - 4x}$ OE e.g. $\frac{e}{2e^{3t}} = \sqrt{1 - x}$ (OE) for M1 , then further correct working needed before printed answer for A1 .			
(c)	From (b) $e^{3t} = \frac{e}{2\sqrt{1-x}}$ $\ln(e^{3t}) = \ln\left(\frac{e}{2\sqrt{1-x}}\right)$ $3t = \ln e - \ln(2\sqrt{1-x})$ $y = \frac{e}{2\sqrt{1-x} [1 - \ln 2\sqrt{1-x}]}$	M1 A1	2	Find $3t$ (or t) in terms of x ; must have used laws of logs correctly on both sides (possibly $\ln e = 1$ at this stage) From $y = \frac{e^{3t}}{3t}$; must be in this form
	Alt. From x , $t = \frac{1}{6}(2 - \ln(4 - 4x))$ M1 or an expression for $3t$ then A1 for printed answer.			
	Total		9	

Q8	Solution	Mark	Total	Comment
(a)	$\theta = \tan^{-1}\left(\frac{3x}{2}\right) \Rightarrow 2 \tan \theta = 3x$ $2 \sec^2 \theta \cdot \frac{d\theta}{dx} = 3$ $\sec^2 \theta = 1 + \tan^2 \theta = 1 + \left(\frac{3x}{2}\right)^2$ $\frac{d\theta}{dx} = \frac{3}{2\left(1+\frac{9x^2}{4}\right)} = \frac{6}{4+9x^2}$	B1 M1 A1	3	or $2\sec^2 \theta = 3 \frac{dx}{d\theta}$ Use of correct identity to get $\sec^2 \theta$ in terms of x; condone missing bracket. Correct algebra with $k = 6$. $\frac{k}{4+9x^2}$ is given so be convinced
	If implicit differentiation not used then 0/3. Embedded answer of $k = 6$ is sufficient. The M1 and A1 marks are available even if the candidate has $\frac{dy}{dx}$ as their derivative.			
(b)	$9y(4+9x^2) \frac{dy}{dx} = \operatorname{cosec} 3y$ $\int 9y \sin 3y \, dy = \int \frac{1}{4+9x^2} \, dx$ LHS: parts with $u = (9)y$ and $dv = \sin 3y$ $(9) \left(-\frac{1}{3} y \cos 3y + \int \frac{1}{3} \cos 3y \, dy \right)$ $-3y \cos 3y + \sin 3y$ RHS: $\frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right)$ $-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + c$ Using $x = 0$ and $y = \frac{\pi}{3}$ to find c $\sin \pi - 3 \cdot \frac{\pi}{3} \cdot \cos \pi = 0 + c \quad \text{gives } c = \pi$ $-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + \pi$	B1 M1 A1 A1 B1ft M1 A1	7	Correct separation including dy and dx at 'ends' of integrals (do not penalise unless the dy or dx is directly under the fraction bar), the integral signs at the 'front' and the integrands. The 9 can be on RHS as $\frac{1}{9}$. M1 for $p y \cos 3y + q \int \cos 3y \, dy$ If 9 used on the LHS ft on k from (a) i.e. $\frac{1}{k} \tan^{-1}\left(\frac{3x}{2}\right)$. Must have an expression of form $p \sin 3y + q y \cos 3y = r \tan^{-1}\left(\frac{3x}{2}\right) + c$ and use $x=0$ and $y = \frac{\pi}{3}$ to find c. PI by a correct ft value for c. OE but must be a complete, correct expression
	Correct separation must be seen on a single line but accept $\int \frac{9y}{\operatorname{cosec} 3y} \, dy$ on LHS. With the 9 on the RHS, the indefinite integral line should read $\frac{1}{9} \sin 3y - \frac{1}{3} y \cos 3y = \frac{1}{54} \tan^{-1}\left(\frac{3x}{2}\right) + k$.			
	Total		10	