AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# A-level MATHEMATICS

## Unit Pure Core 4

Friday 16 June 2017 Afternoon Time allowed: 1 hour 30 minutes

**Materials**
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

| For Examiner's Use | |
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| Question | Mark |
| 1 | |
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| **TOTAL** | |

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** A curve is defined by the parametric equations

$$x = (t-1)^3 \quad , \quad y = 3t - \frac{8}{t^2} \qquad t \neq 0$$

**(a)** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$.

**[3 marks]**

**(b)** Find the equation of the normal at the point on the curve where $t = 2$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**[3 marks]**

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**2 (a)** Express $7\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of $\alpha$ to the nearest $0.1°$.

**[3 marks]**

**(b)** Use your answer to part **(a)** to solve the equation $7\cos 2\theta + 3\sin 2\theta = 5$ in the interval $0° < \theta < 180°$, giving your solutions to the nearest $0.1°$.

**[3 marks]**

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**3 (a)** The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 6x^3 - 11x^2 + 2x + 8$.

**(i)** Use the Factor Theorem to show that $(3x + 2)$ is a factor of $\mathrm{f}(x)$.

**[2 marks]**

**(ii)** Show that $\mathrm{f}(x)$ has no other linear factors.

**[4 marks]**

**(b)** The polynomial $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \mathrm{f}(x) - (6x^2 - 2x - 4)$.

Given that $(3x + 2)$ is a factor of $\mathrm{g}(x)$, express $\mathrm{g}(x)$ as a product of three linear factors.

**[2 marks]**

**(c)** The function h is defined by $\mathrm{h}(x) = \dfrac{\mathrm{g}(x)}{6x^3 - 5x^2 - 6x}$.

Show that $\mathrm{h}(x)$ can be simplified to the form $p + qx^n$ where $p$, $q$ and $n$ are integers.

**[2 marks]**

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**4 (a)** Find the binomial expansion of $(1-4x)^{-\frac{1}{2}}$ up to and including the term in $x^2$.

**[2 marks]**

**(b)** Find the binomial expansion of $(16+4x)^{\frac{3}{4}}$ up to and including the term in $x^2$.

**[3 marks]**

**(c)** Hence find the expansion of $\sqrt{\dfrac{(16+4x)^{\frac{3}{2}}}{(1-4x)}}$ in ascending powers of $x$ up to and including the term in $x^2$.

**[2 marks]**

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**5 (a)** By replacing $3\theta$ by $(2\theta + \theta)$ show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

**[4 marks]**

**(b)** By using the result from part **(a)** and assuming that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(2\sin^3\theta + 3\right) \mathrm{d}\theta$$

**[6 marks]**

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**6** The line $l_1$ has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ -2 \\ p \end{bmatrix}$ where $p$ is an integer.

The line $l_2$ has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$.

The points $A$ and $C$ have coordinates $(3, 1, -1)$ and $(2, 1, -3)$ respectively.

**(a)** The point $A$ lies on $l_1$. Show that $p = 4$.

**[2 marks]**

**(b)** Show that the lines $l_1$ and $l_2$ are perpendicular.

**[1 mark]**

**(c)** Show that the lines $l_1$ and $l_2$ do **not** intersect.

**[3 marks]**

**(d)** The point $B$ lies on $l_1$ such the triangle $ABC$ is isosceles with $AC = BC$.
Find the coordinates of $B$.

**[6 marks]**

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**7** A curve $C$ is defined by the equation

$$\sin 3y + 3e^{-2x}y + 2x^2 = 5$$

**(a)** Find an expression for $\frac{dy}{dx}$.

**[6 marks]**

**(b) (i)** Show that, at the points on $C$ where $\frac{dy}{dx} = 0$, $y = r\,x\,e^{2x}$, where $r$ is a rational number.

**[2 marks]**

**(ii)** Hence show that there is a point on $C$ in the interval $1 < x < 1.2$ where $\frac{dy}{dx} = 0$.

**[4 marks]**

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**8 (a)** It is given that $\dfrac{1}{x(k-x)}$ can be expressed as $A\left(\dfrac{1}{x}+\dfrac{1}{k-x}\right)$ where $A$ and $k$ are positive constants. Find $A$ in terms of $k$.

**[2 marks]**

**(b)** A rumour is spreading through a school of $1200$ pupils. The rate at which the rumour is spreading can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{x(1200-x)}{3600}$$

where $x$ is the number of pupils who have heard the rumour $t$ hours after $11.00\text{ am}$.

By $11.00\text{ am}$, $300$ pupils have heard the rumour. Taking $t=0$ as $11.00\text{ am}$, use integration to solve this differential equation to show that

$$t=3\ln\left(\frac{3x}{1200-x}\right)$$

**[5 marks]**

**(c)** Use this model to:

**(i)** find the time of day by which half of the pupils will have heard the rumour, giving your answer to the nearest $5$ minutes

**[2 marks]**

**(ii)** find $x$ in terms of $t$ and hence find the number of pupils who will have heard the rumour by $3.00\text{ pm}$.

**[3 marks]**

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**END OF QUESTIONS**