

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Unit Pure Core 4

Friday 15 June 2018

Afternoon

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



Answer **all** questions.

Answer each question in the space provided for that question.

- 1 (a)** Express $\frac{10 + 24x - 12x^2}{(3 - x)(1 + 4x)}$ in the form $A + \frac{B}{3 - x} + \frac{C}{1 + 4x}$, where A , B and C are integers.

[4 marks]

- (b)** Hence find $\int_0^2 \frac{10 + 24x - 12x^2}{(3 - x)(1 + 4x)} dx$, giving your answer in the form $p + q \ln 3$.

[5 marks]

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Answer space for question 1



QUESTION
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Answer space for question 1

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2 The angle α is **acute** and $\cos \alpha = \frac{\sqrt{3}}{3}$. The angle β is **obtuse** and $\sin \beta = \frac{1}{3}$.

(a) Show that $\tan \alpha = \sqrt{2}$ and find an exact value for $\tan \beta$.

[3 marks]

(b) Hence show that $\tan(\alpha - \beta)$ can be written as $p\sqrt{2}$, where p is a rational number.

[2 marks]

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3 (a) Find the binomial expansion of $(1 - 9x)^{\frac{2}{3}}$ up to and including the term in x^2 . **[2 marks]**

(b) (i) Find the binomial expansion of $(64 - 9x)^{\frac{2}{3}}$ up to and including the term in x^2 . **[3 marks]**

(ii) Use your expansion from part **(b)(i)** to find an estimate for $67^{\frac{2}{3}}$, giving your answer in the form $p + \frac{q}{r}$ where p , q and r are positive integers with $q < r$. **[2 marks]**

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Answer space for question 3



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4 The polynomial $f(x)$ is defined by $f(x) = 18x^3 - 3x^2 - 28x - 12$.

(a) (i) Use the Factor Theorem to show that $(3x + 2)$ is a factor of $f(x)$.

[2 marks]

(ii) Express $f(x)$ as a product of linear factors.

[3 marks]

(b) The function g is defined for all real values of θ by

$$g(\theta) = 18 \sin 2\theta \cos \theta - 3 \cos 2\theta + 20 \sin \theta + 27$$

(i) Show that the equation $g(\theta) = 0$ can be written as $f(x) = 0$, where $x = \sin \theta$.

[4 marks]

(ii) Hence solve the equation $g(\theta) = 0$, giving your answers, in radians, to three significant figures in the interval $0 \leq \theta \leq 2\pi$.

[2 marks]

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- 5** In a conservation area, a disease is spreading amongst two species of wild animal, P and Q , which is reducing their numbers.

Previous experience has shown that the number of each of the species P and Q can be modelled by

$$p(t) = 4500e^{-\frac{1}{20}t} \quad \text{and} \quad q(t) = 3000e^{-\frac{1}{40}t} \quad \text{respectively}$$

where t is the time in weeks after the disease is first detected.

This outbreak of the disease was first detected on 1 May.

- (a)** Use the two models to find:

- (i)** the number of species P on 1 May;

[1 mark]

- (ii)** the number of species Q after 36 weeks from 1 May, giving your answer to the nearest 10;

[1 mark]

- (iii)** after how many weeks the number of species P will first fall below 1500.

[2 marks]

- (b)** Use logarithms and the two models to calculate the value of t when the number of species Q will be four times that of species P . Give your answer to the nearest whole number.

[3 marks]

- (c)** When $t = T$ the number of species Q first exceeds that of species P by 300.

- (i)** Use this information and the two models to derive a quadratic equation in x where $x = e^{-\frac{1}{40}T}$.

[2 marks]

- (ii)** Hence find the number of days after 1 May when this difference of 300 animals will first occur. Give your answer to the nearest day.

[3 marks]

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6 A curve is defined by the equation

$$\cos 3y + y \sin^2 3x = x + k$$

The point $P\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ lies on this curve.

(a) Find the exact value of the constant k .

[1 mark]

(b) Find an expression for $\frac{dy}{dx}$.

[6 marks]

(c) Find the equation of the tangent to the curve at P , giving your answer in the form $y = mx + c$.

[2 marks]

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7 (a) Using a suitable substitution, or otherwise, find

$$\int \frac{x}{(7 + 2x^2)^2} dx$$

[3 marks]

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{3x e^{4y}}{(7 + 2x^2)^2}$$

given that $y = 0$ when $x = 2$.

Give your answer in the form $y = f(x)$.

[6 marks]

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- 8 The points A and C have coordinates $(3, -1, 2)$ and $(0, -1, -2)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$.

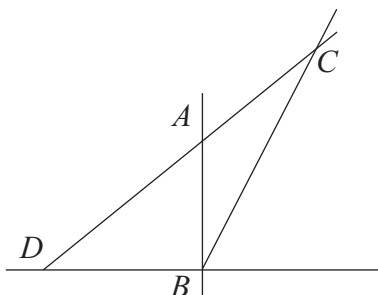
- (a) (i) The point B lies on l where $\lambda = 2$. Find the coordinates of B .

[1 mark]

- (ii) Find the **acute** angle ABC , giving your answer to the nearest 0.1° .

[5 marks]

- (b) The point D lies on a line through C and A such that angle ABD is a right angle.



The point E completes the rectangle $ABDE$. Find the coordinates of E .

[7 marks]

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