

A-LEVEL Mathematics

Mechanics 1B – MM1B Mark scheme

6360 June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aga.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
Α	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
Е	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Do not allow misreads in this question					
Q	Solution	Mark	Total	Comment	
1	$48 \times 1.2 = m \times 16$	M1A1		M1: Seeing 48×1.2 , award if 57.6 seen	
	48×1.2			without any calculation.	
	$m = \frac{48 \times 1.2}{16} = 3.6 \text{ kg}$	A1	3	A1: Correct equation.	
				A1: Correct mass from correct working.	
	Total		3		
				If weight used consistently instead of mass	
				deduct 1 mark.	

Do not allow misreads in this question						
Q	Solution	Mark	Total	Comment		
2. (a)	$V = \sqrt{2^2 + 6^2} = \sqrt{40} = 6.32 \mathrm{m s^{-1}}$	M1A1	2	M1: Equation or expression to find V or V^2 based on Pythagoras. Must have a +. A1: Correct V . Accept AWRT 6.32. Accept $2\sqrt{10}$ or $\sqrt{40}$. Note that just $V^2 = 2^2 + 6^2$ Scores M1A0. OR (if angle found first)		
				M1:Using 2 or 6 with the sin or cos of their angle. A1: Correct <i>V</i> .		
2. (b)	$\tan^{-1}\left(\frac{6}{2}\right) = 71.6^{\circ}$	M1A1		M1: Seeing tan with 6 and 2. (Can be either way round.)		
	$\sin^{-1}\left(\frac{6}{2\sqrt{10}}\right) = 71.6^{\circ}$	(M1A1)	3	A1: Seeing AWRT 72° or 18°. A1: Final answer of 198°. CAO		
	or $\cos^{-1}\left(\frac{2}{2\sqrt{10}}\right) = 71.6^{\circ}$ $\theta = 270 - 71.6 = 198.4^{\circ}$ 198° to nearest degree.	(M1A1) A1		M1: Use of sin or cos with 2 or 6 in the numerator and their answer to (a) as the denominator. A1: Seeing AWRT 72° or 18°. A1: Final answer of 198. CAO		
	OR $\tan^{-1}\left(\frac{2}{6}\right) = 18.4^{\circ}$	(M1A1)		If working in radians, do not award final A1 mark unless converted to degrees. Note that intermediate answers of AWRT 1.25 or AWRT 0.322 score M1A1.		
	$\sin^{-1}\left(\frac{2}{2\sqrt{10}}\right) = 18.4^{\circ}$	(M1A1)				
	or $\cos^{-1}\left(\frac{6}{2\sqrt{10}}\right) = 18.4^{\circ}$ $\theta = 180 + 18.4 = 198.4^{\circ}$	(M1A1)				
	198° to nearest degree.	(A1)	5			

	Do not allow misreads in this question.					
Q	Solution	Mark	Total	Comment		
3. (a)	$T = \frac{100000 \sin 25^{\circ}}{\sin 20^{\circ}} = 124000$	M1A1	3	M1: Resolving perpendicular to the direction of motion. Only award for consistent use of trigonometry as in the following cases: $100000 \sin 25^\circ = \pm T \sin 20^\circ \text{ or } \pm T \cos 70^\circ$ $100000 \cos 65^\circ = \pm T \cos 70^\circ \text{ or } \pm T \sin 20^\circ$ $100000 \sin 65^\circ = \pm T \sin 70^\circ \text{ or } \pm T \cos 20^\circ$ $100000 \cos 25^\circ = \pm T \cos 20^\circ \text{ or } \pm T \sin 70^\circ$ A1: Correct equation. A1: Correct T . Accept 124 kN . Accept AWRT 124000 .		
3. (b)	$100000\cos 25^{\circ} + 123565\cos 20^{\circ} - 20000 = 500000a$ $a = \frac{100000\cos 25^{\circ} + 123565\cos 20^{\circ} - 20000}{500000}$ $a = 0.373 \text{ m s}^{-2}$ OR (Taking opposite direction as positive.) $20000 - 100000\cos 25^{\circ} - 123565\cos 20^{\circ} = 500000a$ $a = \frac{20000 - 100000\cos 25^{\circ} - 123565\cos 20^{\circ}}{500000}$ $a = -0.373 \text{ m s}^{-2}$	M1M1 A1F A1	4	M1: Seeing 500000 <i>a</i> or 500 <i>a</i> anywhere in an equation. May be implied by division. M1: Resultant force (ie LHS in this solution) by resolving parallel to direction of motion. Only award for the following cases, with AWRT 124000 or their answer to part (a): 100000cos 25° + 123565cos 20° – 20000 100000sin 65° + 123565sin 70° – 20000 100000cos 65° + 123565cos 70° – 20000 or with equivalent trigonometry as in part (a). A1F: Correct equation, with AWRT 124000 or their answer to part (a) A1: Correct acceleration, accept AWRT ±0.373 from correct working.		
	Total		7	71000 11 11 11 ±0.57 + 110111 12 +0000		
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	Do not allow misreads in this question.					
Q	Solution	Mark	Total	Comment		
4. (a)	$7\mathbf{i} + 6\mathbf{j} = 4\mathbf{i} + 2\mathbf{j} + 10\mathbf{a}$	M1A1		M1: Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$. Allow if \mathbf{u}		
	$\mathbf{a} = \frac{3\mathbf{i} + 4\mathbf{j}}{10} = (0.3\mathbf{i} + 0.4\mathbf{j}) \text{ m s}^{-2}$	A1	3	substituted for v and v substituted for u after a correct statement of the constant acceleration equation. A1: Correct expression. A1: Correct acceleration.		
4. (b)	$\mathbf{r} = \frac{1}{2}((4\mathbf{i} + 2\mathbf{j}) + (7\mathbf{i} + 6\mathbf{j})) \times 10$ $= 5(11\mathbf{i} + 8\mathbf{j})$	M1A1		M1: Using $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$.		
		A1		A1: Correct expression.		
	= 55 i + 40 j OR			A1: Correct position vector.		
	$\mathbf{r} = (4\mathbf{i} + 2\mathbf{j}) \times 10 + \frac{1}{2}(0.3\mathbf{i} + 0.4\mathbf{j}) \times 10^{2}$	(M1A1)		M1: Using $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$.		
	= 40i + 20j + 15i + 20j = 55i + 40j	(A1)		May have their a from part (a). Must use correct velocity.		
	OR	(A1)		A1: Correct expression.		
	$\mathbf{r} = (7\mathbf{i} + 6\mathbf{j}) \times 10 - \frac{1}{2}(0.3\mathbf{i} + 0.4\mathbf{j}) \times 10^2$	(M1A1)	5	A1: Correct position vector.		
	$= 70\mathbf{i} + 60\mathbf{j} - (15\mathbf{i} + 20\mathbf{j})$ = $55\mathbf{i} + 40\mathbf{j}$	(A1)		dM1: Finding magnitude of their position vector. A1: Correct distance. Accept 68 or AWRT		
	$d = \sqrt{55^2 + 40^2} = 68.0 \mathrm{m}$	dM1A1		$68.0 \text{ or } 5\sqrt{185}$.		
4. (c)	Ave. Velocity = $\frac{55\mathbf{i} + 40\mathbf{j}}{10}$	M1		M1: Their displacement from part (b) divided by 10. A1: Correct average velocity.		
	$= (5.5\mathbf{i} + 4\mathbf{j}) \mathrm{m s}^{-1}$	A1	2	Condone taking means!		
	Total		10			

Do not allow misreads in this question.					
Q	Solution	Mark	Total	Comment	
5. (a)	$2mg - T = 2ma$ $T = 3ma$ $2mg - 3ma = 2ma$ $a = \frac{2g}{5} = (3.92) \text{ m s}^{-2}$	M1A1 B1	4	M1: Three term equation of motion for the particle. Must be either $2mg - T = 2ma$ or $T - 2mg = 2ma$. OE . A1: Correct equation for the particle. B1: Correct equation of motion for the block. Must be consistent with first equation. A1: Correct acceleration. Allow 0.4g oe. Note that use of $g = 9.81$ gives 3.92. SC2: For "whole string method" leading to	
5. (b)	$v^2 = 0^2 + 2 \times 3.92 \times 1.2$ $v = \sqrt{9.408} = 3.07 \text{ m s}^{-1}$	M1A1	3	correct acceleration. Award either 2 or 0 marks. M1: Use of constant acceleration equation with $u = 0$, $s = 1.2$ and their numerical value for a from part (a). A1: Correct equation. A1: Correct speed. AWRT 3.07.	
5. (c)	$2mg - T = 2ma$ $T - F = 3ma$ $T - 0.8 \times 3mg = 3ma$ $-0.4mg = 5ma$ $a = -0.08g = -0.784 \text{ m s}^{-2}$	B1 B1 M1A1	5	B1: Three term equation of motion for the particle. Must be either $2mg - T = 2ma$ or $T - 2mg = 2ma$ OE. B1: Seeing $F = 0.8 \times 3mg$ OE. M1: Three term equation of motion for the block. Must be either $T - F = 3ma$ or $F - T = 3ma$ OE. A1: Correct equation for the block. Equation must be consistent with other equation. A1: Correct acceleration, sign consistent with working. Accept -0.08 g oe. Accept -0.785 from $g = 9.81$. SC3: For "whole string method" leading to correct acceleration. Award either 3 or 0	
5. (d)	$v^{2} = 9.408 + 2 \times (-0.784) \times 0.9$ $v = \sqrt{7.9968} = 2.83 \mathrm{m s^{-1}}$	M1A1	3	marks. M1: Use of constant acceleration equation with their answers to parts (b) and (c) with $s = 0.9$. A1: Correct equation. A1: Correct speed. AWRT 2.83. Note use of $g = 9.81$ gives 2.83.	

5	i. (e)	If the size of the block is not negligible there will be mixed friction on the block as it passes from the smooth to rough sections of the surface.	B1	1	B1: Statement about issue of moving from smooth to rough.
		Total		16	

	Do not allow mis			
Q	Solution	Mark	Total	Comment
6. (a)	$0.5 = 8\sin 30^{\circ}t - 4.9t^{2} + 1.2$ $4.9t^{2} - 4t - 0.7 = 0$ $t = -0.148 \text{ or } 0.964$ Require $t = 0.964$	M1A1 A1 dM1 A1	5	M1: Seeing 8 sin 30° t or 8 cos30° t and $\pm 4.9t^2$ and 0.5 or 1.2 or 0.7. A1: Correct terms with possible sign errors. A1: Correct equation.
	Time Up = 0.40816 Time Down = 0.40816 + 0.14812 = 0.55628 Total Time = 0.40816 + 0.55628 = 0.964 s	(M2A1 A1A1)	5	Solving their Quadratic If working shown in full (ie use of quadratic equation formula): dM1: At least one solution seen and no more than one substitution error in formula. A1: Correct solution selected. AWRT 0.964 If working not shown in full (ie values obtained direct from calculator): dM1: Obtaining at least one correct solution to the quadratic equation. A1: Showing the two correct solutions and selecting the positive one. AWRT 0.964 Note that use of g = 9.81 gives 0.964.
	OR			M2: Method to find total time, by adding 2 or 3 times. A1: Correct time up. AWRT 0.41 A1: Correct time down. AWRT 0.56 A1: Correct total time. AWRT 0.964
	$v^{2} = (8 \sin 30^{\circ})^{2} + 2 \times 9.8 \times 0.7$ $v = -5.4516$ $-5.4516 = 8 \sin 30^{\circ} - 9.8t$ $t = 0.964$ or $5.4516 = -8 \sin 30^{\circ} + 9.8t$ $t = 0.964$ or $-0.7 = \frac{1}{2} (8 \sin 30^{\circ} - 5.4516)t$ $t = 0.964$	(M1A1 A1) (dM1) (A1)		M1: Using two constant acceleration equations to find <i>t</i> . Allow 8sin30° or 8cos30°. A1: Correct first equation. A1: Seeing AWRT ±5.45 dM1: Correct second equation. A1: AWRT 0.964

	h = 0.5			
6. (d)		B1	1	B1: CAO
	$v_y^2 = (8\sin 30^\circ)^2 + 2 \times (-9.8) \times (-0.7)$ $v_y^2 = 29.72$ $v_y = 5.45$ OR $v_y = 8\sin 30^\circ - 9.8 \times 0.964$ $= -5.45$ OR $-0.7 = 0.964v_y + 4.9 \times 0.964^2$ $v_y = -5.45$ OR $-0.7 = \frac{1}{2}(8\sin 30^\circ + v_y) \times 0.964$	M1 A1 (M1) (A1)		M1: Using constant acceleration equation(s) to find the vertical component of the velocity, including ±0.7, 8 sin 30° and g. A1: Correct vertical component. AWRT 5.4 or 5.5.
	$v_y = -5.45$ $v = \sqrt{29.72 + (8\cos 30^\circ)^2} = 8.82 \text{ m s}^{-1}$ Total	dM1 A1	4	dM1: Finding speed using horizontal component as 8cos30°. A1: Correct speed. AWRT 8.82 Accept 8.81.

	Do not allow misreads in this question.					
Q	Solution	Mark	Total	Comment		
7. (a)	40 N 80 N 490 N	B2	2	 B2: Five forces shown, with arrow heads and labels. Accurate Labels for example: R or N, but not mg oe. 490 or W or mg or 50g. F, but not μR. Award B1if one error or one missing force. Condone addition of components provided a significantly different notation is used. 		
7. (b)	$40\sin 20^{\circ} + 80\sin 30^{\circ} + R = 490$ $R = 436 \text{ N}$	M1A1 A1	3	M1: Four term equation (or three term expression for <i>R</i>) with ±490 or ±50 <i>g</i> , and consistent use of trig one of the following: 40 sin 20° + 80 sin 30° 40 cos 70° + 80 cos 60° 40 cos 20° + 80 cos 30° 40 sin 70° + 80 sin 60° A1: Correct equation or expression. A1: Correct normal reaction. AWRT 436. Accept AWRT 437 using g =9.81.		
7. (c)	$F \le 0.6 \times 436.32$ $F \le 262 \text{ N}$ OR $F = 0.6 \times 436.32$ F = 262 N $80 \cos 30^{\circ} - 40 \cos 20^{\circ} = 31.7 \text{ N}$ Remains at rest as $31.7 < 262$.	M1 A1 (M1) (A1) dM1 dM1 A1	5	M1: Use of $F \le \mu R$ or $F = \mu R$ (or $F \ge \mu R$) with 0.6 and their R from part (b). A1: Correct maximum friction. AWRT 262. dM1: Seeing $80\cos 30^\circ$ dM1: Seeing $40\cos 20^\circ$ A1: Correct conclusion, with a reasonable justification. That is remains at rest. CSO		
7. (d)	If the crate is modelled as a particle then any tendency to rotate is not considered. Total	B1	1 11	B1: Comment about potential for rotation.		

Do not allow misreads in this question.					
Q	Solution	Mark	Total	Comment	
8. (a)	$s = \frac{1}{2} \times 4 \times 8 = 16 \text{ m}$ \mathbf{OR}	M1A1		M1: Finding the area of the triangle formed by the two lines and the <i>v</i> -axis. A1: Correct distance.	
	$s_A = \frac{1}{2}(1+5.8) \times 8 = 27.2 \text{ m}$ $s_B = \frac{1}{2}(5+5.8) \times 8 = 43.2 \text{ m}$ 43.2 - 27.2 = 16 m <i>B</i> has travelled further.	(M1A1) B1	3	M1: Areas of two trapezia to find distance travelled by each train. A1: Subtracting to find the correct distance. B1: Stating that <i>B</i> has travelled further, not necessarily supported by correct numeric arguments.	
8. (b)	$a_A = \frac{5.8 - 1}{8} = 0.6 \mathrm{m s^{-2}}$	B1		B1: Correct acceleration of <i>A</i> . B1: Correct acceleration of <i>B</i> .	
	$a_B = \frac{5.8 - 5}{8} = 0.1 \mathrm{m s^{-2}}$	B1		B1: Correct expression for the displacement of <i>A</i> .	
	$s_A = t + 0.3t^2$ $s_B = 5t + 0.05t^2$	B1 B1		B1: Correct expression for the displacement of <i>B</i> .	
	$t + 0.3t^{2} - (5t + 0.05t^{2}) = 9$ $0.25t^{2} - 4t - 9 = 0$ $t^{2} - 16t - 36 = 0$	M1 A1		M1: Difference for their two quadratic displacements equated to 9. A1: Correct equation may be unsimpilified.	
	(t-18)(t+2) = 0 t = 18 or $t = -2t = 18$	dM1 A1	8	Solving their Quadratic If working shown in full (eg factorising): dM1: Award for correct factorisation or $(t+18)(t-2) = 0$ A1: Correct solution stated.	
				If working not shown in full (ie values obtained direct from calculator): dM1: Obtaining at least one correct solution to the quadratic equation. A1: Showing the two correct solutions and selecting the positive one.	
	Total		11		
	TOTAL		75		