AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Decision 2

Tuesday 25 June 2019 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** below shows an activity diagram for a construction project. The time needed for each activity is given in days.

(a) Find the earliest start time and latest finish time for each activity and insert their values on **Figure 1**.

[4 marks]

(b) Find the critical paths and state the minimum time for completion of the project.

[3 marks]

(c) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as early as possible.

[3 marks]

(d) A delay in supplies means that Activity I takes 11 days instead of 4.

(i) Determine the effect on the earliest possible starting times for activities K and L.

[2 marks]

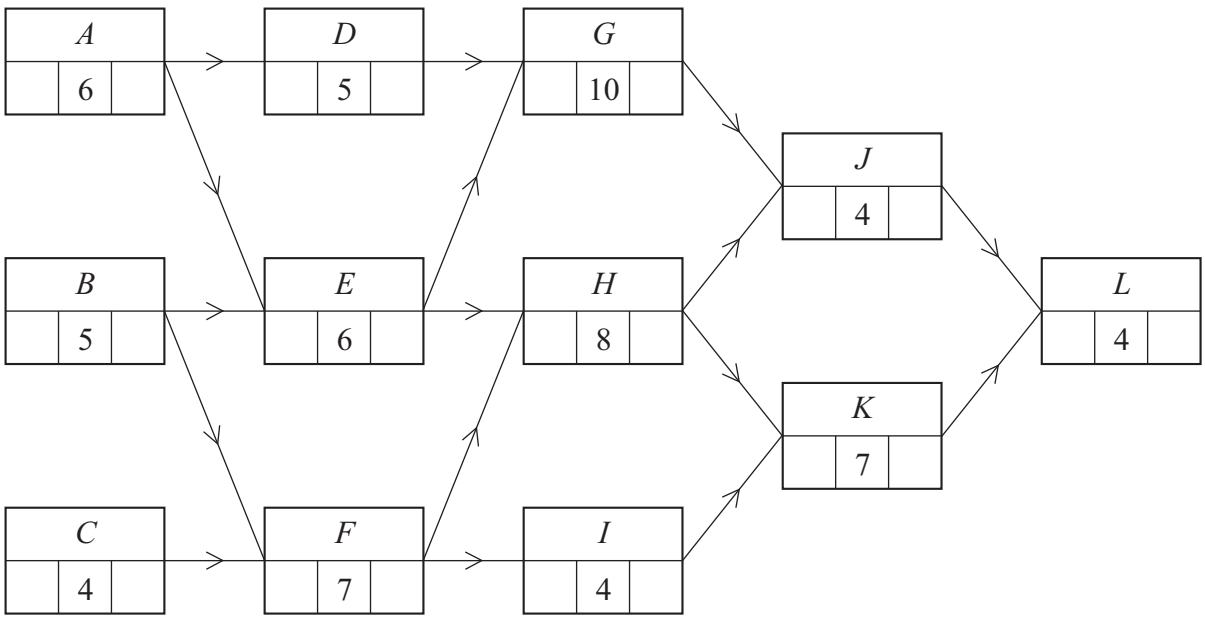
(ii) State the number of days by which the completion of the project is now delayed.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

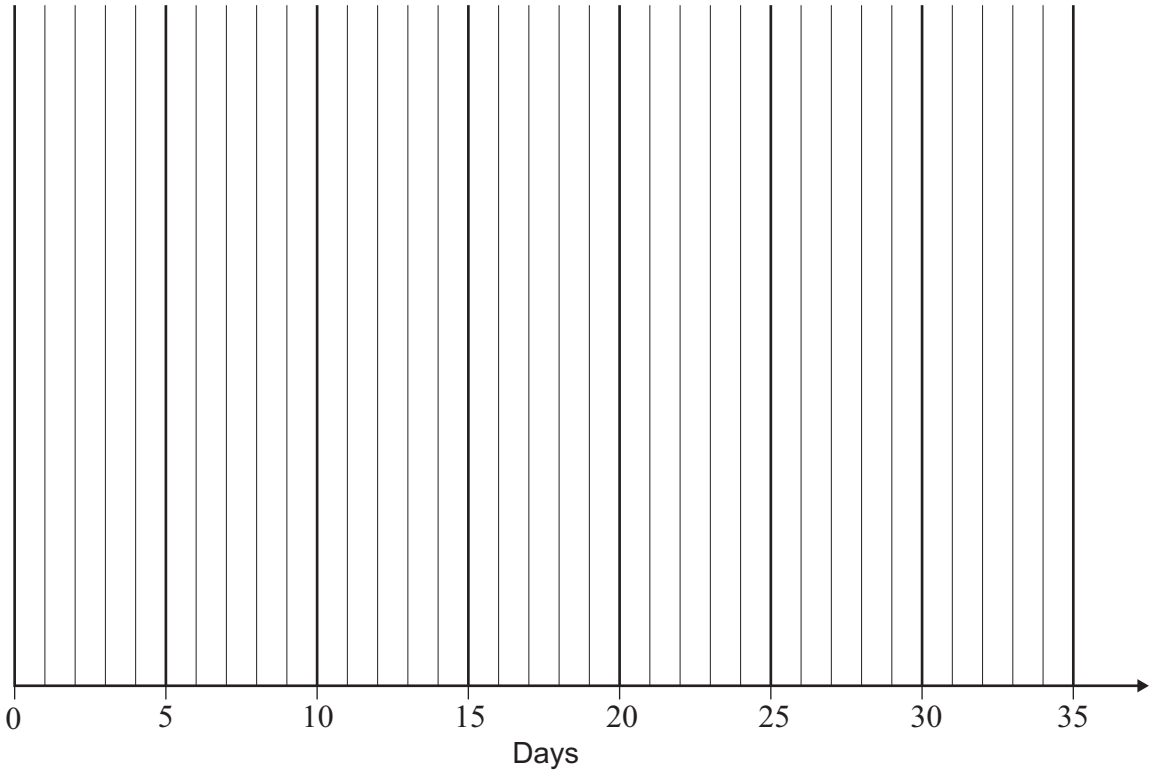
Figure 1



QUESTION PART REFERENCE

Answer space for question 1

Figure 2



2 A farmer has five fields. He intends to grow a different crop in each of four fields and to leave one of the fields unused. The farmer tests the soil in each field and calculates a score for growing each of the four crops. The scores are given in the table below.

	Field A	**Field B**	**Field C**	**Field D**	**Field E**
Crop 1	32	24	16	36	28
Crop 2	40	30	16	32	24
Crop 3	18	20	24	34	24
Crop 4	36	22	34	30	38

The farmer's aim is to maximise the total score for the four crops.

(a) (i) Modify the table of values by first subtracting each value in the table above from 40 and then adding an extra row of equal values.

[1 mark]

(ii) Explain why the Hungarian algorithm can now be applied to the new table of values to maximise the total score for the four crops.

[3 marks]

(b) (i) By reducing rows first, show that the table from part **(a)(i)** becomes

4	12	20	0	p
0	10	24	8	16
16	14	10	0	q
2	16	4	8	0
0	0	0	0	0

State the values of the constants p and q.

[2 marks]

(ii) Show that the zeros in the table from part **(b)(i)** can be covered by one horizontal and three vertical lines, and use the Hungarian algorithm to decide how the four crops should be allocated to the fields.

[6 marks]

(iii) Hence find the maximum possible total score for the four crops.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

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3 (a) Two people, Raj and Cal, play a zero-sum game. The game is represented by the following pay-off matrix for Raj.

		Cal		
	Strategy	***X***	***Y***	***Z***
Raj	**I**	-8	7	-6
	II	5	1	-2
	III	-3	3	-4

Show that this game has a stable solution and state the play-safe strategy for each player.

[4 marks]

(b) Ros and Carly play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros, where x is a constant.

		Carly	
	Strategy	C_1	C_2
Ros	R_1	4	x
	R_2	-3	3

Ros chooses strategy R_1 with probability p.

(i) Find expressions for the expected gains for Ros when Carly chooses each of the strategies C_1 and C_2.

[2 marks]

(ii) Given that the value of the game is $\frac{9}{11}$, find the value of p and the value of x.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

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4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 6y + kz$, where k is a constant.

The initial Simplex tableau is given below.

P	x	y	z	s	t	u	value
1	-2	-6	$-k$	0	0	0	0
0	5	3	10	1	0	0	15
0	7	6	4	0	1	0	28
0	4	3	6	0	0	1	12

(a) In addition to $x \geqslant 50$, $y \geqslant 50$, $z \geqslant 50$, write down three inequalities involving x, y and z for this problem.

[2 marks]

(b) (i) By choosing the first pivot from the y-column, perform one iteration of the Simplex method.

[4 marks]

(ii) Given that the optimal value has not been reached, find the possible values of k.

[2 marks]

(c) In the case when $k = 20$:

(i) perform one further iteration;

[4 marks]

(ii) interpret the final tableau and state the values of the slack variables.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

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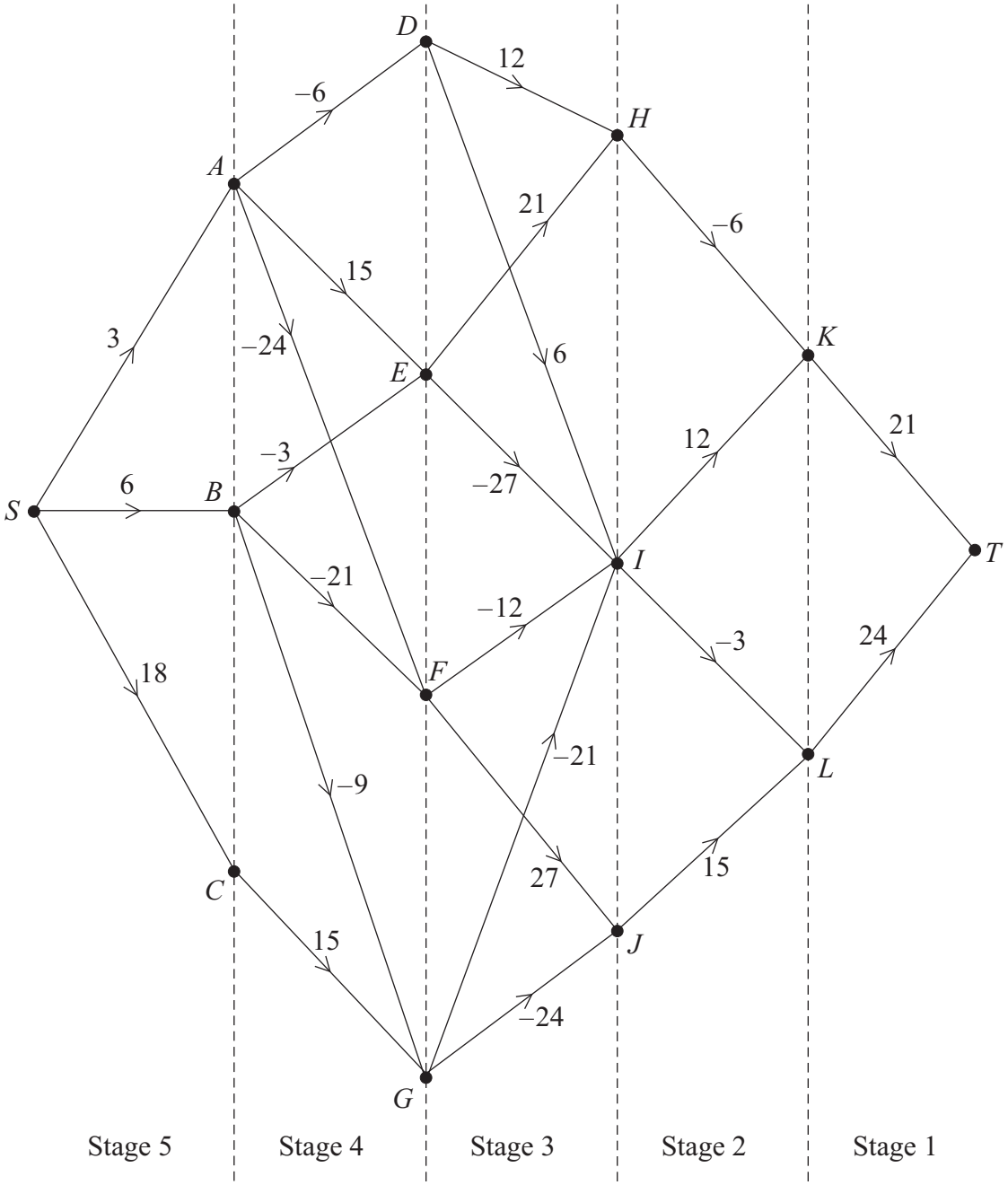
QUESTION PART REFERENCE	**Answer space for question 4**

5 A company has a number of stores. The following network shows the possible actions and profits over the next five years. The number on each edge is the expected profit, in millions of pounds. A negative number indicates a loss due to investment in new stores.



(a) Working backwards from T, use dynamic programming to maximise the expected profits over the five years. You must complete the table on **Figure 3** on the opposite page as your solution.

[7 marks]

(b) State the maximum expected profit and the sequence of vertices from S to T in order to achieve this.

[2 marks]

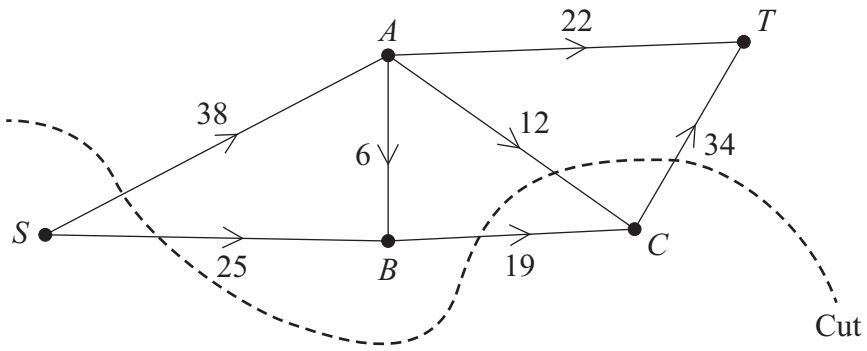
QUESTION PART REFERENCE

Answer space for question 5

Figure 3

Stage	State	From	Value	
1	K	T		

6 (a) The network shows a flow from S to T along a system of pipes, with the capacity, in litres per minute, indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 97.

[1 mark]

(ii) The cut shown on the diagram can be represented as $\{S, C\}$, $\{A, B, T\}$. Complete the table on **Figure 4** opposite, giving the value of each of the 8 possible cuts.

[4 marks]

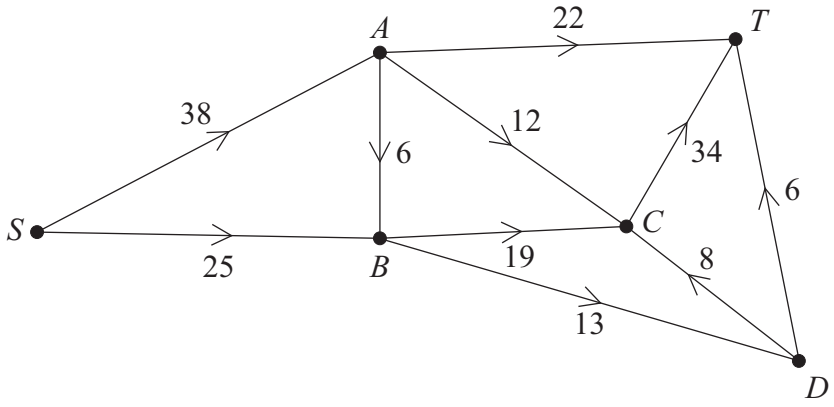
(iii) State the value of the maximum flow through the network, giving a reason for your answer.

[2 marks]

(iv) Indicate on **Figure 5** opposite, a possible flow along each edge corresponding to this maximum flow.

[2 marks]

(b) Extra pipes, BD, CD and DT, are added to form a new system, with the capacity, in litres per minute, indicated on each edge of the network below.



(i) Taking your values from **Figure 5** as the initial flow, use the labelling procedure on **Figure 6** opposite, to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network.

[4 marks]

(ii) State the value of the new maximum flow, and, on **Figure 7** opposite, indicate a possible flow along each edge corresponding to this maximum flow.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

Figure 4

Cut		Value
$\{S\}$	$\{A, B, C, T\}$	63
$\{S, A\}$	$\{B, C, T\}$	
$\{S, B\}$	$\{A, C, T\}$	
$\{S, C\}$	$\{A, B, T\}$	97
$\{S, A, B\}$	$\{C, T\}$	53
$\{S, A, C\}$	$\{B, T\}$	53
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	

Figure 5

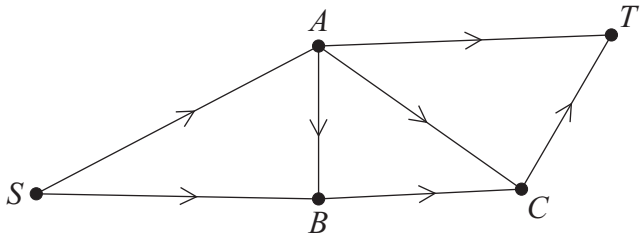
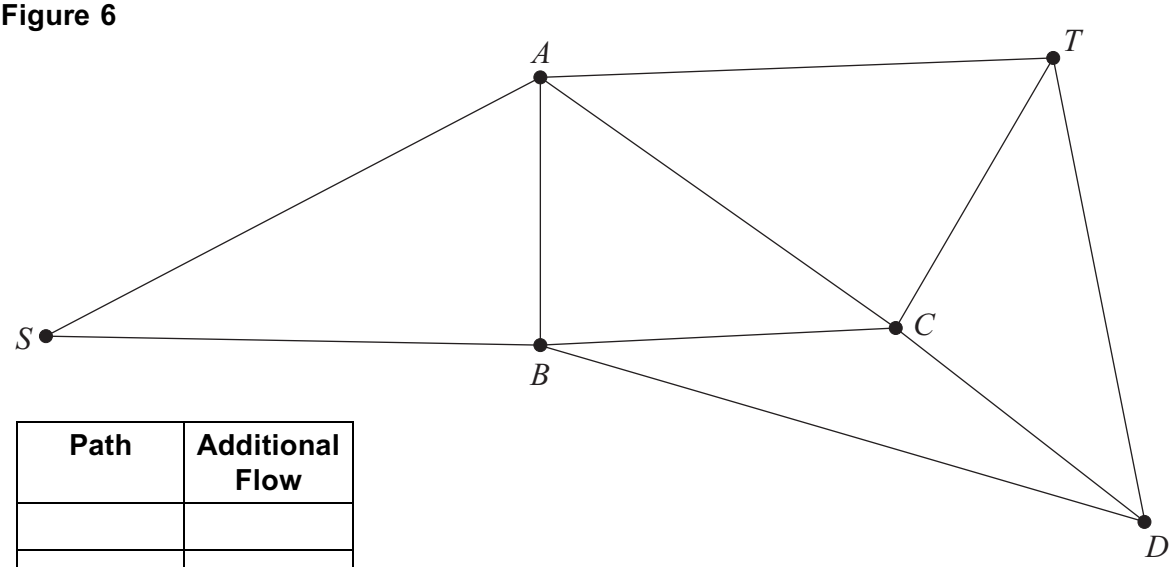
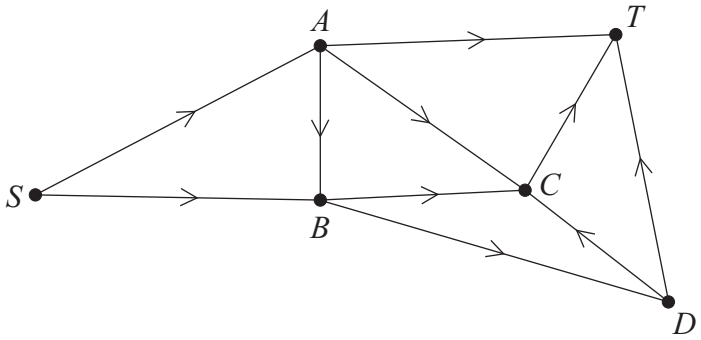


Figure 6



Path	Additional Flow

Figure 7



New maximum flow is _____

Turn over ▶

QUESTION PART REFERENCE	**Answer space for question 6**

END OF QUESTIONS