

SENIOR KANGAROO

Friday 29 November 2019

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England & Wales: Year 13 or below

Scotland: S6 or below

Northern Ireland: Year 14 or below

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.

2. Time allowed: **60 minutes**.

No answers, or personal details, may be entered after the allowed time is over.

3. The use of blank or lined paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.

4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.

Pay close attention to the example on the Answer Sheet that shows how to code your answers.

5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. **Scoring rules:**

5 marks are awarded for each correct answer;

There is no penalty for giving an incorrect answer.

7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Senior Kangaroo should be sent to:

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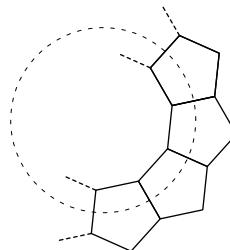
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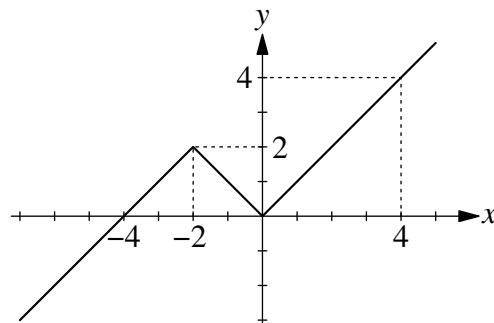
- What is the sum of all the factors of 144?
- When I noticed that $2^4 = 4^2$, I tried to find other pairs of numbers with this property. Trying 2 and 16, I realised that 2^{16} is larger than 16^2 . How many times larger is 2^{16} ?
- The two diagonals of a quadrilateral are perpendicular. The lengths of the diagonals are 14 and 30. What is the area of the quadrilateral?
- The integer n satisfies the inequality $n + (n + 1) + (n + 2) + \cdots + (n + 20) > 2019$. What is the minimum possible value of n ?

- Identical regular pentagons are arranged in a ring. The partially completed ring is shown in the diagram. Each of the regular pentagons has a perimeter of 65. The regular polygon formed as the inner boundary of the ring has a perimeter of P . What is the value of P ?



- For natural numbers a and b we are given that $2019 = a^2 - b^2$. It is known that $a < 1000$. What is the value of a ?
- How many positive? integers n exist such that both $\frac{n+1}{3}$ and $3n+1$ are three-digit integers?
- The function $J(x)$ is defined by:

$$J(x) = \begin{cases} 4 + x & \text{for } x \leq -2, \\ -x & \text{for } -2 < x \leq 0, \\ x & \text{for } x > 0. \end{cases}$$



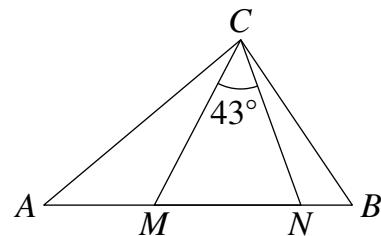
How many distinct real solutions has the equation $J(J(J(x))) = 0$?

- What is the smallest three-digit number K which can be written as $K = a^b + b^a$, where both a and b are one-digit positive integers?
- What is the value of $\sqrt{13 + \sqrt{28 + \sqrt{281}}} \times \sqrt{13 - \sqrt{28 + \sqrt{281}}} \times \sqrt{141 + \sqrt{281}}$?

11. In the triangle ABC the points M and N lie on the side AB such that $AN = AC$ and $BM = BC$.

We know that $\angle MCN = 43^\circ$.

Find the size in degrees of $\angle ACB$.



12. What is the value of $A^2 + B^3 + C^5$, given that:

$$A = \sqrt[3]{16\sqrt{2}}$$

$$B = \sqrt{9\sqrt[3]{9}}$$

$$C = [(\sqrt[5]{2})^2]^2$$

13. The real numbers a and b , where $a > b$, are solutions to the equation $3^{2x} - 10 \times 3^{x+1} + 81 = 0$. What is the value of $20a^2 + 18b^2$?

14. A number N is the product of three distinct primes. How many distinct factors does N^5 have?

15. Five Bunchkins sit in a horizontal field. No three of the Bunchkins are sitting in a straight line. Each Bunchkin knows the four distances between her and each of the others. Each Bunchkin calculates and then announces the total of these distances. These totals are 17, 43, 56, 66 and 76. A straight line is painted joining each pair of Bunchkins. What is the total length of paint required?

16. The real numbers x and y satisfy the equations:

$$xy - x = 180 \quad \text{and} \quad y + xy = 208.$$

Let the two solutions be (x_1, y_1) and (x_2, y_2) .

What is the value of $x_1 + 10y_1 + x_2 + 10y_2$?

17. In triangle ABC , $\angle BAC$ is 120° . The length of AB is 123. The point M is the midpoint of side BC . The line segments AB and AM are perpendicular. What is the length of side AC ?

18. An integer is said to be *chunky* if it consists only of non-zero digits by which it is divisible when written in base 10.

For example, the number 936 is Chunky since it is divisible by 9, 3 and 6.

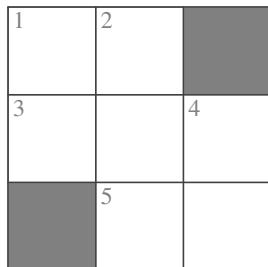
How many chunky integers are there between 13 and 113?

19. The square $ABCD$ has sides of length 105. The point M is the midpoint of side BC . The point N is the midpoint of BM . The lines BD and AM meet at the point P . The lines BD and AN meet at the point Q .

What is the area of triangle APQ ?

20. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.

What is the answer to 3 ACROSS?



ACROSS

1. A composite factor of 1001
3. Not a palindrome
5. pq^3 where p, q prime and $p \neq q$

DOWN

1. One more than a prime, one less than a prime
2. A multiple of 9
4. p^3q using the same p, q as 5 ACROSS