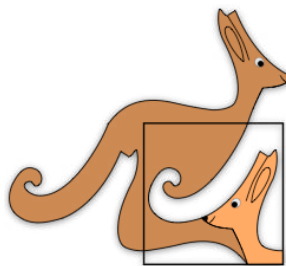


United Kingdom  
Mathematics Trust



# THE ANDREW JOBBINGS SENIOR KANGAROO

**25 – 26 November 2020**

Organised by the United Kingdom Mathematics Trust

*a member of the Association Kangourou sans Frontières*



*England & Wales: Year 13 or below*

*Scotland: S6 or below*

*Northern Ireland: Year 14 or below*

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.  
No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.  
Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**  
5 marks are awarded for each correct answer;  
There is no penalty for giving an incorrect answer.
7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

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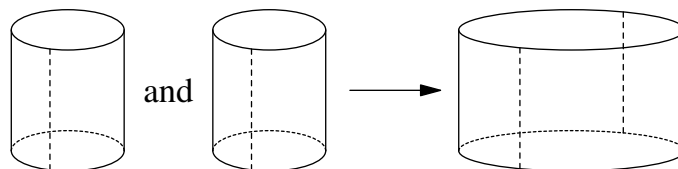
1. What is the difference between the greatest and the least of the following five quantities?

$$20 + 20 \quad 20 \times 20 \quad 202 + 0 \quad (2^0)^{(2^0)} \quad 20 + 2 + 0$$

2. The positive integers  $x$  and  $y$  satisfy the equation  $yx^2 + xy^2 = 70$ . What is the value of  $x^4 + y^4$ ?

3. How many distinct integer solutions  $(x, y)$  are there to the equation  $5^1 + 4^2 + 3^3 + 2^4 = x^y$ ?

4. Two identical cylindrical sheets are cut open along the dotted lines and glued together to form one bigger cylindrical sheet, as shown. The smaller sheets each enclose a volume of 100. What volume is enclosed by the larger sheet?



5. Let  $a^b = \frac{1}{8}$ . What is the value of  $a^{-3b}$ ?

6. For what value of  $x$  does the expression  $x^2 - 600x + 369$  take its minimum value?

7. Margot writes the numbers 1,2,3,4,5,6,7 and 8 in the top row of a table, as shown. In the second row she plans to write the same set of numbers, in any order.

Each number in the third row is obtained by finding the sum of the two numbers above it.

In how many different ways can Margot complete row 2 so that every entry in row 3 is even?

1	2	3	4	5	6	7	8

8. The number  $(2^{222})^5 \times (5^{555})^2$  is  $Q$  digits long. What is the largest prime factor of  $Q$ ?

9. The radii of two concentric circles are in the ratio 1 : 3.

$AC$  is a diameter of the larger circle.  $BC$  is a chord of the larger circle and is tangent to the smaller circle.  $AB$  has length 140.

What is the radius of the larger circle?

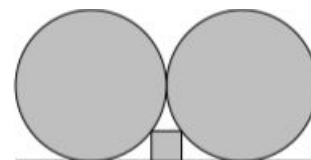
10. What is the smallest 3-digit positive integer  $N$  such that  $2^N + 1$  is a multiple of 5?

11. A circle is drawn inside a regular hexagon so that it touches all six sides of the hexagon. The area of the circle is  $\pi \times 64\sqrt{3}$ . What is the area of the hexagon?

12. What is the value of  $\sqrt{20212020 \times 20202021 - 20212021 \times 20202020}$ ?

13. How many ordered triples of positive integers  $(x, y, z)$  satisfy  $(x^y)^z = 1024$ ?

14. Let  $a$ ,  $b$ ,  $c$  and  $d$  be distinct positive integers such that  $a + b$ ,  $a + c$  and  $a + d$  are all odd and are all square. Let  $L$  be the least possible value of  $a + b + c + d$ . What is the value of  $10L$ ?
15. On an island, kangaroos are always either grey or red. One day, the number of grey kangaroos increased by 28% while the number of red kangaroos decreased by 28%. The ratios of the two types of kangaroos were exactly reversed. By what percentage did the total number of kangaroos change?
16. A square fits snugly between the horizontal line and two touching circles of radius 1000, as shown. The line is tangent to the circles. What is the side-length of the square?
17. How many solutions does equation  $|||x - 1| - 1| - 1| = 1$  have?  
*The modulus function  $|x|$  evaluates the absolute value of a number; for example  $|6| = |-6| = 6$ .*
18. The operation  $\diamond$  is defined on two positive whole numbers as the number of distinct prime factors of the product of the two numbers. For example  $8 \diamond 15 = 3$ .  
 What is the cube of the value of  $(720 \diamond 1001)$ ?
19. A random number generator gives outputs of 1, 2, 3, 4 and 5 with equal probability. The values of  $a$ ,  $b$  and  $c$  are each chosen by running the generator once. The probability that  $a \times b + c$  is even can be written as a fraction in its lowest terms as  $\frac{N}{D}$ .  
 What is the value of  $10N + D$ ?
20. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.  
 What is the answer to 3 ACROSS?



1	2	
3		4
	5	

## ACROSS

1. A multiple of 7
3. The answer to this Question
5. More than 10

## DOWN

1. A multiple of a square of an odd prime; neither a square nor a cube
2. The internal angle of a regular polygon; the exterior angle is between  $10^\circ$  and  $20^\circ$
4. A proper factor of 5 ACROSS but not a proper factor of 1 DOWN