

## SENIOR 'KANGAROO' MATHEMATICAL CHALLENGE

**Friday 2nd December 2016**

**Organised by the United Kingdom Mathematics Trust**

*The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.*

### RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.

Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.

5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

*Enquiries about the Senior Kangaroo should be sent to:*

*Maths Challenges Office, School of Maths Satellite,*

*University of Leeds, Leeds, LS2 9JT*

*Tel. 0113 343 2339*

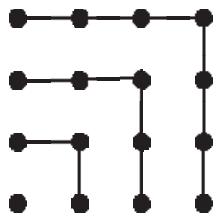
*[www.ukmt.org.uk](http://www.ukmt.org.uk)*

1. Using this picture we can observe that

$$1 + 3 + 5 + 7 = 4 \times 4.$$

What is the value of

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21?$$



2. Both rows of the following grid have the same sum. What is the value of \* ?

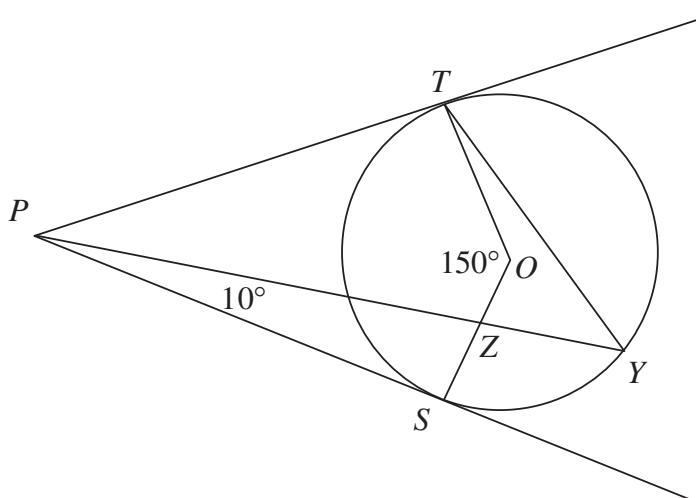
1	2	3	4	5	6	7	8	9	10	1050
11	12	13	14	15	16	17	18	19	20	*

3. Andrew has two containers for carrying water. The containers are cubes without tops and have base areas of  $4 \text{ dm}^2$  and  $36 \text{ dm}^2$  respectively. Andrew has to completely fill the larger cube with pond water, which must be carried from the pond using the smaller cube. What is the smallest number of visits Andrew has to make to the pond with the smaller cube?

4. How many four-digit numbers formed only of odd digits are divisible by five?

5. The notation  $|x|$  is used to denote the absolute value of a number, regardless of sign. For example,  $|7| = |-7| = 7$ . The graphs  $y = |2x| - 3$  and  $y = |x|$  are drawn on the same set of axes. What is the area enclosed by them?

6.



In the diagram,  $PT$  and  $PS$  are tangents to a circle with centre  $O$ . The point  $Y$  lies on the circumference of the circle; and the point  $Z$  is where the line  $PY$  meets the radius  $OS$ .

Also,  $\angle SPZ = 10^\circ$  and  $\angle TOS = 150^\circ$ .

How many degrees are there in the sum of  $\angle PTY$  and  $\angle PYT$ ?

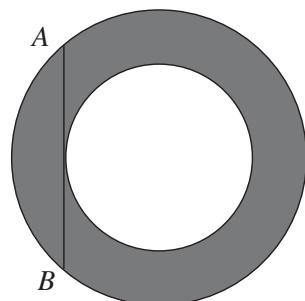
7. Bav is counting the edges on a particular prism. The prism has more than 310 edges, it has fewer than 320 edges and its number of edges is odd. How many edges does the prism have?

8. The real numbers  $x$ ,  $y$  and  $z$  are a solution  $(x, y, z)$  of the equation  $(x^2 - 9)^2 + (y^2 - 4)^2 + (z^2 - 1)^2 = 0$ . How many different possible values are there for  $x + y + z$ ?

9. The diagram shows two concentric circles. Chord  $AB$  of the larger circle is tangential to the smaller circle.

The length of  $AB$  is 32 cm and the area of the shaded region is  $k\pi \text{ cm}^2$ .

What is the value of  $k$ ?



10. Consider the expression  $1 * 2 * 3 * 4 * 5 * 6$ .

Each star in the expression is to be replaced with either '+' or 'x'.

$N$  is the largest possible value of the expression. What is the largest prime factor of  $N$ ?

11. Stephanie enjoys swimming. She goes for a swim on a particular date if, and only if, the day, month (where January is replaced by '01' through to December by '12') and year are all of the same parity (that is they are all odd, or all are even). On how many days will she go for a swim in the two-year period between January 1st of one year and December 31st of the following year inclusive?

12. Delia is joining three vertices of a square to make four right-angled triangles.

She can create four triangles doing this, as shown.

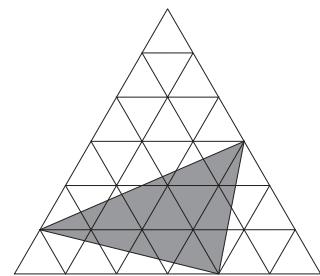


How many right-angled triangles can Delia make by joining three vertices of a regular polygon with 18 sides?

13. This year, 2016, can be written as the sum of two positive integers  $p$  and  $q$  where  $2p = 5q$  (as  $2016 = 1440 + 576$ ). How many years between 2000 and 3000 inclusive have this property?

14. The lengths of the sides of a triangle are the integers 13,  $x$ ,  $y$ . It is given that  $xy = 105$ . What is the length of the perimeter of the triangle?

15. The large equilateral triangle shown consists of 36 smaller equilateral triangles. Each of the smaller equilateral triangles has area  $10 \text{ cm}^2$ . The area of the shaded triangle is  $K \text{ cm}^2$ . Find  $K$ .



16. A function  $f(x)$  has the property that, for all positive  $x$ ,  $3f(x) + 7f\left(\frac{2016}{x}\right) = 2x$ . What is the value of  $f(8)$ ?

17. Students in a class take turns to practise their arithmetic skills. Initially a board contains the integers from 1 to 10 inclusive, each written ten times. On each turn a student first deletes two of the integers and then writes on the board the number that is one more than the sum of those two deleted integers. Turns are taken until there is only one number remaining on the board. Assuming no student makes a mistake, what is the remaining number?

18. The sum of the squares of four consecutive positive integers is equal to the sum of the squares of the next three consecutive integers. What is the square of the smallest of these integers?

19. Erin lists all three-digit primes that are 21 less than a square. What is the mean of the numbers in Erin's list?

20. A barcode of the type shown in the two examples is composed of alternate strips of black and white, where the leftmost and rightmost strips are always black. Each strip (of either colour) has a width of 1 or 2. The total width of the barcode is 12. The barcodes are always read from left to right. How many distinct barcodes are possible?

