

STEP Support Programme

STEP 2 Calculus Questions

1 2005 S2 Q1

Find the three values of x for which the derivative of $x^2e^{-x^2}$ is zero.

Given that a and b are distinct positive numbers, find a polynomial $P(x)$ such that the derivative of $P(x)e^{-x^2}$ is zero for $x = 0$, $x = \pm a$ and $x = \pm b$, but for no other values of x .

2 2006 S2 Q4

By making the substitution $x = \pi - t$, show that

$$\int_0^\pi xf(\sin x)dx = \frac{1}{2}\pi \int_0^\pi f(\sin x)dx,$$

where $f(\sin x)$ is a given function of $\sin x$.

Evaluate the following integrals:

(i) $\int_0^\pi \frac{x \sin x}{3 + \sin^2 x} dx;$

(ii) $\int_0^{2\pi} \frac{x \sin x}{3 + \sin^2 x} dx;$

(iii) $\int_0^\pi \frac{x |\sin 2x|}{3 + \sin^2 x} dx.$

3 2007 S2 Q6

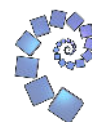
(i) Differentiate $\ln(x + \sqrt{3 + x^2})$ and $x\sqrt{3 + x^2}$ and simplify your answers.

Hence find $\int \sqrt{3 + x^2} dx$.

(ii) Find the two solutions of the differential equation

$$3 \left(\frac{dy}{dx} \right)^2 + 2x \frac{dy}{dx} = 1$$

that satisfy $y = 0$ when $x = 1$.



4 2008 S2 Q7

- (i) By writing $y = u(1 + x^2)^{\frac{1}{2}}$, where u is a function of x , find the solution of the equation

$$\frac{1}{y} \frac{dy}{dx} = xy + \frac{x}{1 + x^2}$$

for which $y = 1$ when $x = 0$.

- (ii) Find the solution of the equation

$$\frac{1}{y} \frac{dy}{dx} = x^2 y + \frac{x^2}{1 + x^3}$$

for which $y = 1$ when $x = 0$.

- (iii) Give, without proof, a conjecture for the solution of the equation

$$\frac{1}{y} \frac{dy}{dx} = x^{n-1} y + \frac{x^{n-1}}{1 + x^n}$$

for which $y = 1$ when $x = 0$, where n is an integer greater than 1.

If you are finding question 4 quite tricky you might like to tackle the question below first (which uses the same ideas but is a STEP I question).

5 2010 S1 Q6

Show that, if $y = e^x$, then

$$(x - 1) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0. \quad (*)$$

In order to find other solutions of this differential equation, now let $y = ue^x$, where u is a function of x . By substituting this into (*), show that

$$(x - 1) \frac{d^2 u}{dx^2} + (x - 2) \frac{du}{dx} = 0. \quad (**)$$

By setting $\frac{du}{dx} = v$ in (**) and solving the resulting first order differential equation for v , find u in terms of x . Hence show that $y = Ax + Be^x$ satisfies (*), where A and B are any constants.

