# STEP Support Programme

## STEP 2 Complex Numbers: Hints

**1** For the first three parts you can use what you know about combining arguments and moduli and use diagrams to help you sketch the loci.

Alternatively, you can substitute $z = x + \mathrm{i}y$ into the given expressions and hence find an equation in $x$ and $y$.

For the last part, a clear diagram is essential. Use what you know about arguments, and also what you know about cyclic quadrilaterals.

**2** For the first part, expand the brackets and then use some Trig formulae to simplify the real and imaginary parts. Then use proof by induction. The result $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$ is known as *de Moivre's theorem* and is required knowledge for STEP III.

For the next part, start by expanding $(5-\mathrm{i})^2(1+\mathrm{i})$. Use the result $\arg(z_1 \times z_2) = \arg z_1 + \arg z_2$. You can find the arguments of the complex numbers involved by drawing a diagram and using $\tan\theta = \frac{opp}{adj}$.

For the last part you are not told what to consider. Comparing the result given here to the one given earlier should help you find some useful complex numbers to consider.

**3** For the "stem" substitute $z = x + iy$ into the given expression for $w$ and simplify. To deal with the complex denominator multiply top and bottom by the complex conjugate of the denominator.

The resulting expressions for $u$ and $v$ will be useful throughout the rest of the question.

**(i)** Substitute the two given expressions for $x$ and $y$ into the results for $u$ and $v$.

Another way of writing the circle equation $x^2 + y^2 = 1$ is to use a parameter and take $x = \cos\alpha$ and $y = \sin\alpha$.

There are values of $\alpha$ for which $\tan\alpha$ is undefined.

**(ii)** This is very similar to the previous part. Work out how the restrictions on $x$ relate to restrictions on $\theta$.

**(iii)** Substitute $x = 0$ and see what happens.

**(iv)** This looks a little like part **(i)**, so try a substitution for $x$ similar to that in part **(i)**. You might have to fiddle a bit with it to make things cancel nicely.

**4** Draw a diagram showing $O$, $A$ and $C$. Remember that $|a-c|$ represents the distance between $A$ and $C$, and $|a-c|^2 = (a-c)\,(a^*-c^*)$. By Pythagoras' Theorem a triangle is right-angled if and only if ...

For the next part you can find the equation for a circle centre $C$ with a known radius (and in this case $r^2 = |a-c|^2$). You should expect to have to use the condition that $\angle OAC = 90°$ at some point. Point $P$ (or $P'$) lies on the circle if and only if it satisfies the equation of the circle.

For the last part the equation of the circle will be the same, but you cannot use the condition $2aa^* = ac^* + ca^*$ to simplify the equation, in fact this is what you are trying to prove!

Substituting for $P$ and $P'$ gives two equations which you need to manipulate to show that $2aa^* = ac^* + ca^*$, or perhaps more usefully $2aa^* - ac^* - ca^* = 0$.