

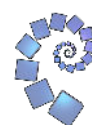
STEP Support Programme

STEP 2 Trigonometry Questions: Hints

- 1 Start by using $\cos(3x) = \cos(2x + x)$. Do something very similar for $\sin(3x)$.
 - (i) Rewrite $\sin^3 x$ in terms of $\sin x$ and $\sin 3x$. There is one value of α which must give $\int_0^\alpha f(x) dx = 0$ from which you can find the value of $c = \cos(\alpha)$. The question is a “write down” which implies that not much work is needed.
 - (ii) Eustace believes the given statement for $n = 1$, so he will even get $\int \sin x dx$ wrong. You will want to write Eustace’s attempt in terms of c . There is one obvious value of α (and therefore c) for which Eustace will get the correct value, which can be used to help find the other values.

You are asked to find **all** the values, of which there are infinitely many! Refer to the “general solutions” part of the topic notes.

- 2 There are many ways to prove the result in the “stem”. You could start on the left hand side or the right hand side.
 - (i) First find the value of x which will give $\frac{1}{4}\pi - \frac{1}{2}x = \frac{1}{8}\pi$. Then you can write $\frac{11}{24}\pi$ in terms of $\frac{1}{8}\pi$ and another useful fraction of π .
 - (ii) Since you are given both sides you can instead show an equivalent statement to be true. Remember to show all your working!
 - (iii) It looks like the previous work should be useful! Try using $x = \frac{11}{24}\pi$ in (*).



- 3** Note that $\arctan x = \tan^{-1} x$. This is an integration by substitution question, and you will need to use a relationship between $\tan \theta$ and $\sec \theta$.
- (i) (a) Try to find a substitution which will convert I into something of the same form as the “stem” result. You will find that $a = 1$.
- (b) Quite a lot of manipulation of trigonometric functions is needed here, but you do know what you are aiming for. Completing the square might be a useful technique to consider. Remember that $\sin 2A = 2 \sin A \cos A$.
- (ii) This is very similar to part (i)(b). Start with the same substitution as in the previous part and then use a second one to write the integral in the same form as in the stem.
- 4** This is quite a long question with lots of things to think about.
- (i) The first thing that springs to mind is using $\sin 4\theta = 2 \sin 2\theta \cos 2\theta$, but this won’t help find the actual values of θ . You will need this technique later in this part to find the value of $\sin 18^\circ$.
- Start by using $\cos \alpha = \sin(90^\circ - \alpha)$ or $\sin \alpha = \cos(90^\circ - \alpha)$. Remember that if $\cos \alpha = \cos \beta$ it does not mean that $\alpha = \beta$, you could have $\alpha = -\beta$ etc.
- (ii) Start by writing the equation in terms of $s = \sin x$ which will be a quadratic in s^2 . The previous part might be useful.
- (iii) “Hence” means that you need to use some of the previous parts. Try to find a value of 5α for which the equation had the same form as in part (ii).

