STEP Support Programme

STEP 2 Trigonometry Questions

1 2010 S2 Q2

Prove that

$$\cos 3x = 4\cos^3 x - 3\cos x\,.$$

Find and prove a similar result for $\sin 3x$ in terms of $\sin x$.

(i) Let

$$\mathrm{I}(\alpha) = \int_0^{\alpha} \left(7\sin x - 8\sin^3 x\right)\mathrm{d}x\,.$$

Show that

$$\mathrm{I}(\alpha) = -\tfrac{8}{3}c^3 + c + \tfrac{5}{3}\,,$$

where $c = \cos\alpha$. Write down one value of c for which $\mathrm{I}(\alpha) = 0$.

(ii) Eustace believes that

$$\int \sin^n x\,\mathrm{d}x = \frac{\sin^{n+1} x}{n+1}$$

for $n = 1,\ 2,\ 3, \ldots$. Show that Eustace would obtain the correct value of $\mathrm{I}(\beta)$, where $\cos\beta = -\frac{1}{6}$.

Find all values of α for which he would obtain the correct value of $\mathrm{I}(\alpha)$.

2 2009 S2 Q3

Prove that

$$\tan\left(\tfrac{1}{4}\pi - \tfrac{1}{2}x\right) \equiv \sec x - \tan x\,. \qquad (*)$$

(i) Use $(*)$ to find the value of $\tan\frac{1}{8}\pi$. Hence show that

$$\tan\tfrac{11}{24}\pi = \frac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}-\sqrt{6}+1}\,.$$

(ii) Show that

$$\frac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}-\sqrt{6}+1} = 2+\sqrt{2}+\sqrt{3}+\sqrt{6}\,.$$

(iii) Use $(*)$ to show that

$$\tan\tfrac{1}{48}\pi = \sqrt{16+10\sqrt{2}+8\sqrt{3}+6\sqrt{6}}\ -2-\sqrt{2}-\sqrt{3}-\sqrt{6}\,.$$

3 2007 S2 Q3

By writing $x = a\tan\theta$, show that, for $a \neq 0$, $\displaystyle\int \frac{1}{a^2+x^2}\,\mathrm{d}x = \frac{1}{a}\arctan\frac{x}{a} + \text{constant}\,.$

(i) Let $\displaystyle I = \int_0^{\frac{1}{2}\pi} \frac{\cos x}{1+\sin^2 x}\,\mathrm{d}x\,.$

(a) Evaluate I.

(b) Use the substitution $t = \tan\frac{1}{2}x$ to show that $\displaystyle\int_0^1 \frac{1-t^2}{1+6t^2+t^4}\,\mathrm{d}t = \tfrac{1}{2}I\,.$

(ii) Evaluate $\displaystyle\int_0^1 \frac{1-t^2}{1+14t^2+t^4}\,\mathrm{d}t\,.$

4 2011 S2 Q4

(i) Find all the values of θ, in the range $0^\circ < \theta < 180^\circ$, for which $\cos\theta = \sin 4\theta$. Hence show that

$$\sin 18^\circ = \frac{1}{4}\left(\sqrt{5} - 1\right).$$

(ii) Given that

$$4\sin^2 x + 1 = 4\sin^2 2x\,,$$

find all possible values of $\sin x$, giving your answers in the form $p + q\sqrt{5}$ where p and q are rational numbers.

(iii) Hence find two values of α with $0^\circ < \alpha < 90^\circ$ for which

$$\sin^2 3\alpha + \sin^2 5\alpha = \sin^2 6\alpha\,.$$