# STEP Support Programme

## STEP 2 Vectors Questions: Hints

**1** Start by writing $m_{3/4}$ as a general 3D vector. Then use the dot product, for example $m_1 \cdot m_3 = |m_1||m_3| \cos\left(\frac{\pi}{4}\right)$. You are expecting two solutions (one being $m_3$ and one being $m_4$).

Remember that, for example, the direction $1\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ is the same as the direction $4\mathbf{i}+8\mathbf{j}+12\mathbf{k}$ (or any other multiple). You may like to pick a (non-zero) value of one component (but just check that this component cannot be zero first).

For parts **(i)** and **(ii)** it is helpful to find $A$, $B$, $P$ and $Q$ first.

**(i)** If vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular then $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$.

**(ii)** Try and find where they intersect and then show that this solution is not possible.

**2** Start by drawing a clear diagram! Try also to work out what is happening geometrically, what do you know about $OB$ and $OC$? How does $OA$ relate to these? How does $BC$ relate to $OA$?

The point $D$ is found in a very similar way to point $C$, so you can use your previous work to help here without retracing your steps entirely. You need to find $\mu$ in terms of $\mathbf{a}$ and $\mathbf{b}$ (not least because it makes things easier later on!).

If three points are collinear, then the vector between any two of them is a multiple between the vector between one of these and the third one.

**3** Start by drawing a clear diagram showing points $A$, $B$ and $C$, making sure that the three points do not all lie on the same straight line.

It is very useful to note that the equation of the straight line through $X$ and $Y$ has the form $\mathbf{r} = t\mathbf{x} + (1-t)\mathbf{y}$ and the value of $t$ determines where on the line $XY$ a particular point is (it might not be in-between $X$ and $Y$). You could try substituting some values of $\lambda$ and $\mu$ in order to get a feel for what is happening (make sure your values are in the given range).

You may also find it helpful to rewrite the expressions for $\mathbf{p}$ and $\mathbf{q}$, such as $\mathbf{p} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b})$.

You will need to find an equation for the line $PQ$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ (start by finding it in terms of $\mathbf{p}$ and $\mathbf{q}$).

For the last bit, rewrite $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ so that there are two position vectors on each side.

**4** The "Stem" needs an application of the scalar product.

**(i)** If $L_1$ makes the same angle with $OA$ and $OB$ you can use the scalar product to find an equation involving $m$, $n$ and $p$. There are lots of possible lines making the same angle with $OA$ and $OB$ as we are in three dimensions.

If $L_1$ is the angle bisector of $\angle AOB$ then you know what angle $L_1$ makes with $OA$.

**(ii)** This is very similar to the previous part, and you might be able to use some of the working again (with different letters). This time we know what angle $L_2$ makes with $OA$, but there is no restriction on the angle it makes with $OB$.