

## STEP Support Programme

### STEP 2 Vectors Questions: Hints

- 1** Start by writing  $m_{3/4}$  as a general 3D vector. Then use the dot product, for example  $m_1 \cdot m_3 = |m_1||m_3|\cos\left(\frac{\pi}{4}\right)$ . You are expecting two solutions (one being  $m_3$  and one being  $m_4$ ).

Remember that, for example, the direction  $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$  is the same as the direction  $4\mathbf{i} + 8\mathbf{j} + 12\mathbf{k}$  (or any other multiple). You may like to pick a (non-zero) value of one component (but just check that this component cannot be zero first).

For parts **(i)** and **(ii)** it is helpful to find  $A$ ,  $B$ ,  $P$  and  $Q$  first.

**(i)** If vectors  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular then  $\mathbf{a} \cdot \mathbf{b} = 0$ .

**(ii)** Try and find where they intersect and then show that this solution is not possible.

- 2** Start by drawing a clear diagram! Try also to work out what is happening geometrically, what do you know about  $OB$  and  $OC$ ? How does  $OA$  relate to these? How does  $BC$  relate to  $OA$ ?

The point  $D$  is found in a very similar way to point  $C$ , so you can use your previous work to help here without retracing your steps entirely. You need to find  $\mu$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (not least because it makes things easier later on!).

If three points are collinear, then the vector between any two of them is a multiple between the vector between one of these and the third one.

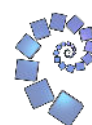
- 3** Start by drawing a clear diagram showing points  $A$ ,  $B$  and  $C$ , making sure that the three points do not all lie on the same straight line.

It is very useful to note that the equation of the straight line through  $X$  and  $Y$  has the form  $\mathbf{r} = t\mathbf{x} + (1 - t)\mathbf{y}$  and the value of  $t$  determines where on the line  $XY$  a particular point is (it might not be in-between  $X$  and  $Y$ ). You could try substituting some values of  $\lambda$  and  $\mu$  in order to get a feel for what is happening (make sure your values are in the given range).

You may also find it helpful to rewrite the expressions for  $\mathbf{p}$  and  $\mathbf{q}$ , such as  $\mathbf{p} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b})$ .

You will need to find an equation for the line  $PQ$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  (start by finding it in terms of  $\mathbf{p}$  and  $\mathbf{q}$ ).

For the last bit, rewrite  $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$  so that there are two position vectors on each side.



4 The “Stem” needs an application of the scalar product.

- (i) If  $L_1$  makes the same angle with  $OA$  and  $OB$  you can use the scalar product to find an equation involving  $m$ ,  $n$  and  $p$ . [There are lots of possible lines making the same angle with  \$OA\$  and  \$OB\$  as we are in three dimensions.](#)

If  $L_1$  is the angle bisector of  $\angle AOB$  then you know what angle  $L_1$  makes with  $OA$ .

- (ii) This is very similar to the previous part, and you might be able to use some of the working again (with different letters). This time we know what angle  $L_2$  makes with  $OA$ , but there is no restriction on the angle it makes with  $OB$ .

