

STEP Support Programme

STEP 2 Vectors Questions

1 2002 S2 Q7

In 3-dimensional space, the lines m_1 and m_2 pass through the origin and have directions $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$, respectively. Find the directions of the two lines m_3 and m_4 that pass through the origin and make angles of $\pi/4$ with both m_1 and m_2 . Find also the cosine of the acute angle between m_3 and m_4 .

The points A and B lie on m_1 and m_2 respectively, and are each at distance $\lambda\sqrt{2}$ units from O . The points P and Q lie on m_3 and m_4 respectively, and are each at distance 1 unit from O . If all the coordinates (with respect to axes \mathbf{i} , \mathbf{j} and \mathbf{k}) of A , B , P and Q are non-negative, prove that:

- (i) there are only two values of λ for which AQ is perpendicular to BP ;
- (ii) there are no non-zero values of λ for which AQ and BP intersect.

2 2011 S2 Q5

The points A and B have position vectors \mathbf{a} and \mathbf{b} with respect to an origin O , and O , A and B are non-collinear. The point C , with position vector \mathbf{c} , is the reflection of B in the line through O and A . Show that \mathbf{c} can be written in the form

$$\mathbf{c} = \lambda\mathbf{a} - \mathbf{b}$$

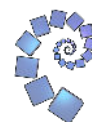
where $\lambda = \frac{2\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}}$.

The point D , with position vector \mathbf{d} , is the reflection of C in the line through O and B . Show that \mathbf{d} can be written in the form

$$\mathbf{d} = \mu\mathbf{b} - \lambda\mathbf{a}$$

for some scalar μ to be determined.

Given that A , B and D are collinear, find the relationship between λ and μ . In the case $\lambda = -\frac{1}{2}$, determine the cosine of $\angle AOB$ and describe the relative positions of A , B and D .



3 2009 S2 Q8

The non-collinear points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} , respectively. The points P and Q have position vectors \mathbf{p} and \mathbf{q} , respectively, given by

$$\mathbf{p} = \lambda\mathbf{a} + (1 - \lambda)\mathbf{b} \quad \text{and} \quad \mathbf{q} = \mu\mathbf{a} + (1 - \mu)\mathbf{c}$$

where $0 < \lambda < 1$ and $\mu > 1$. Draw a diagram showing A , B , C , P and Q .

Given that $CQ \times BP = AB \times AC$, find μ in terms of λ , and show that, for all values of λ , the line PQ passes through the fixed point D , with position vector \mathbf{d} given by $\mathbf{d} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$. What can be said about the quadrilateral $ABDC$?

4 2010 S2 Q5

The points A and B have position vectors $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $5\mathbf{i} - \mathbf{j} - \mathbf{k}$, respectively, relative to the origin O . Find $\cos 2\alpha$, where 2α is the angle $\angle AOB$.

- (i) The line L_1 has equation $\mathbf{r} = \lambda(m\mathbf{i} + n\mathbf{j} + p\mathbf{k})$. Given that L_1 is inclined equally to OA and to OB , determine a relationship between m , n and p . Find also values of m , n and p for which L_1 is the angle bisector of $\angle AOB$.
- (ii) The line L_2 has equation $\mathbf{r} = \mu(u\mathbf{i} + v\mathbf{j} + w\mathbf{k})$. Given that L_2 is inclined at an angle α to OA , where $2\alpha = \angle AOB$, determine a relationship between u , v and w .

Hence describe the surface with Cartesian equation $x^2 + y^2 + z^2 = 2(yz + zx + xy)$.

