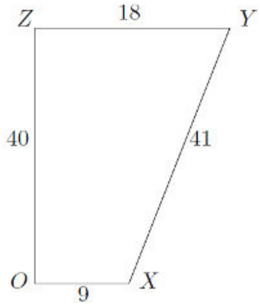
STEP Support Programme

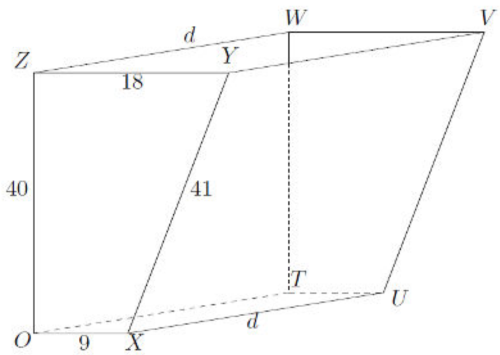
STEP 2 Mechanics Questions

1 2009 S2 Q9

(i) A uniform lamina $OXYZ$ is in the shape of the trapezium shown in the diagram. It is right-angled at O and Z, and OX is parallel to YZ. The lengths of the sides are given by $OX = 9\,\text{cm}$, $XY = 41\,\text{cm}$, $YZ = 18\,\text{cm}$ and $ZO = 40\,\text{cm}$. Show that its centre of mass is a distance $7\,\text{cm}$ from the edge OZ.



(ii) The diagram shows a tank with no lid made of thin sheet metal. The base $OXUT$, the back $OTWZ$ and the front $XUVY$ are rectangular, and each end is a trapezium as in part **(i)** . The width of the tank is $d\,\text{cm}$.



Show that the centre of mass of the tank, when empty, is a distance

$$\frac{3(140 + 11d)}{5(12 + d)}\,\text{cm}$$

from the back of the tank.

The tank is then filled with a liquid. The mass per unit volume of this liquid is k times the mass per unit area of the sheet metal. In the case $d = 20$, find an expression for the distance of the centre of mass of the filled tank from the back of the tank.

2 **2010 S2 Q11**

A uniform rod AB of length $4L$ and weight W is inclined at an angle θ to the horizontal. Its lower end A rests on a fixed support and the rod is held in equilibrium by a string attached to the rod at a point C which is $3L$ from A. The reaction of the support on the rod acts in a direction α to AC and the string is inclined at an angle β to CA. Show that

$$\cot\alpha = 3\tan\theta + 2\cot\beta\,.$$

Given that $\theta = 30^\circ$ and $\beta = 45^\circ$, show that $\alpha = 15^\circ$.

3 **2007 S2 Q11**

In this question take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$ *and neglect air resistance.*

The point O lies in a horizontal field. The point B lies $50\,\mathrm{m}$ east of O. A particle is projected from B at speed $25\,\mathrm{m\,s^{-1}}$ at an angle $\arctan\frac{1}{2}$ above the horizontal and in a direction that makes an angle 60° with OB; it passes to the north of O.

(i) Taking unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ in the directions east, north and vertically upwards, respectively, find the position vector of the particle relative to O at time t seconds after the particle was projected, and show that its distance from O is

$$5(t^2 - \sqrt{5}t + 10)\,\mathrm{m}.$$

When this distance is shortest, the particle is at point P. Find the position vector of P and its horizontal bearing from O.

(ii) Show that the particle reaches its maximum height at P.

(iii) When the particle is at P, a marksman fires a bullet from O directly at P. The initial speed of the bullet is $350\,\mathrm{m\,s^{-1}}$. Ignoring the effect of gravity on the bullet show that, when it passes through P, the distance between P and the particle is approximately $3\,\mathrm{m}$.

4 **2011 S2 Q9**

Two particles, A of mass $2m$ and B of mass m, are moving towards each other in a straight line on a smooth horizontal plane, with speeds $2u$ and u respectively. They collide directly. Given that the coefficient of restitution between the particles is e, where $0 < e \leqslant 1$, determine the speeds of the particles after the collision.

After the collision, B collides directly with a smooth vertical wall, rebounding and then colliding directly with A for a second time. The coefficient of restitution between B and the wall is f, where $0 < f \leqslant 1$. Show that the velocity of B after its second collision with A is

$$\tfrac{2}{3}(1-e^2)u - \tfrac{1}{3}(1-4e^2)fu$$

towards the wall and that B moves towards (not away from) the wall for all values of e and f.

5 **2008 S1 Q9**

This was an old STEP I question which includes content from the new (2019 onwards) STEP II specification.

Two identical particles P and Q, each of mass m, are attached to the ends of a diameter of a light thin circular hoop of radius a. The hoop rolls without slipping along a straight line on a horizontal table with the plane of the hoop vertical. Initially, P is in contact with the table. At time t, the hoop has rotated through an angle θ. Write down the position at time t of P, relative to its starting point, in cartesian coordinates, and determine its speed in terms of a, θ and $\dot{\theta}$. Show that the total kinetic energy of the two particles is $2ma^2\dot{\theta}^2$.

Given that the only external forces on the system are gravity and the vertical reaction of the table on the hoop, show that the hoop rolls with constant speed.

6 2000 S1 Q10

This was an old STEP I question which includes content from the new (2019 onwards) STEP II specification.

Three particles P_1, P_2 and P_3 of masses m_1, m_2 and m_3 respectively lie at rest in a straight line on a smooth horizontal table. P_1 is projected with speed v towards P_2 and brought to rest by the collision. After P_2 collides with P_3, the latter moves forward with speed v. The coefficients of restitution in the first and second collisions are e and e', respectively. Show that

$$e' = \frac{m_2 + m_3 - m_1}{m_1}.$$

Show that $2m_1 \geqslant m_2 + m_3 \geqslant m_1$ for such collisions to be possible.

If m_1, m_3 and v are fixed, find, in terms of m_1, m_3 and v, the largest and smallest possible values for the final energy of the system.

It is important to think of a good way of naming the velocities of the particles before and after each collision; this of course must be carefully set out in your answer.

Don't forget, when you are trying to derive the inequalities, that $0 \leqslant e \leqslant 1$.

7 2008 S3 Q10

This is an old STEP III question, which is now on the STEP II specification

A long string consists of n short light strings joined together, each of natural length ℓ and modulus of elasticity λ. It hangs vertically at rest, suspended from one end. Each of the short strings has a particle of mass m attached to its lower end. The short strings are numbered 1 to n, the nth short string being at the top. By considering the tension in the rth short string, determine the length of the long string. Find also the elastic energy stored in the long string.

A uniform heavy rope of mass M and natural length L_0 has modulus of elasticity λ. The rope hangs vertically at rest, suspended from one end. Show that the length, L, of the rope is given by

$$L = L_0\left(1 + \frac{Mg}{2\lambda}\right),$$

and find an expression in terms of L, L_0 and λ for the elastic energy stored in the rope.